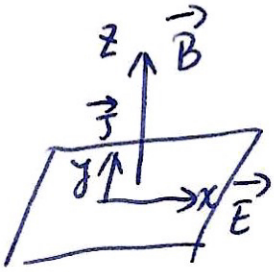
Lecture 3. Quantum Hall Effect.

1. Hall Effect.



$$J_y = \sigma_{yx} E_x$$

2. Landau levels.

$$\nabla \times A = B$$

$$\vec{B} = B_0 (0, 0, 1)$$

$$\vec{A} = (0, B_0 x, 0)$$ → 保持 P_y 为好量子数.

$$H = \sum_{k, p_\alpha, p_\beta} C^{+}_{k p_\alpha} H_{p_\alpha p_\beta}(k) C_{k p_\beta}$$

轨道/自旋指标.

$$H_{p_\alpha p_\beta}(\vec{k}) \rightarrow H_{p_\alpha p_\beta}\left(\vec{k} + \frac{e\vec{A}}{\hbar}\right).$$

↑

Pierels 替换.

类比: $\hat{H} = \frac{(\vec{p} + e\vec{A})^2}{2m} + U(r).$ / $(\vec{p} \leftrightarrow \hbar\vec{k}).$

朗道能级.

μ. 电子填充的最高能量,
下面全满,
无法有净运动.

电流算符:

$$\vec{J} = q\cdot\vec{v}$$
$$= -e\vec{v}$$

$$\vec{v} = -\frac{i}{\hbar}[r, H]$$
$$= \frac{\vec{p}+e\vec{A}}{m}$$
$$= \frac{1}{e}\frac{\partial H}{\partial A}$$

$$\vec{J} = -\frac{\partial H}{\partial \vec{A}}$$

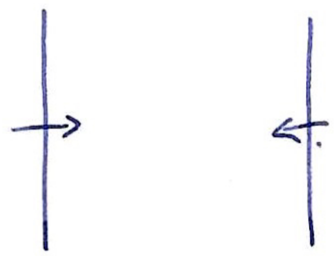
可由哈密顿量对矢势偏导获得.

Feynman - Hellman 定理.

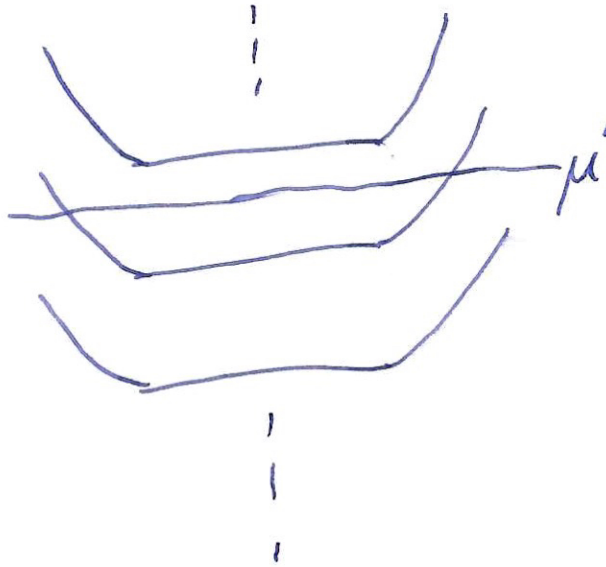
$$\langle \vec{J} \rangle = -\frac{\partial \langle H \rangle}{\partial \vec{A}} = -\frac{\partial E}{\partial \vec{A}}.$$

3. 固体中的朗道能级.

变化1. 边界.



边界受力使能级上弯

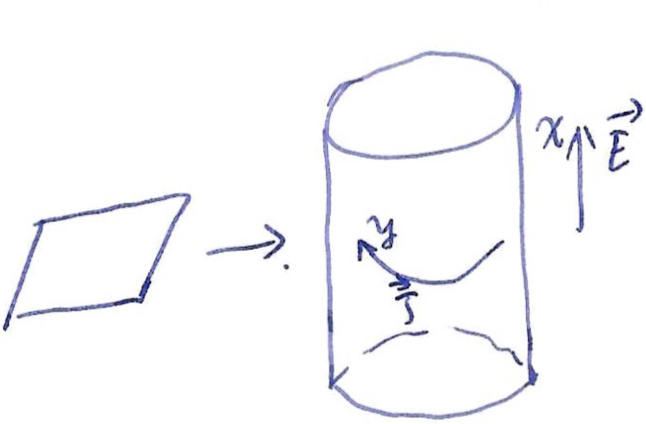


电子填充的最高能量.

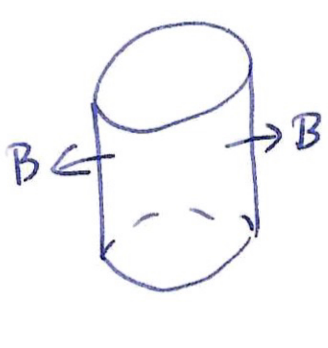
边界不满，可以有净运动.

4. 量子化.

周期性边条件



$\vec{B}$ 原为z方向,
现变为径向. (保持与材料垂直)

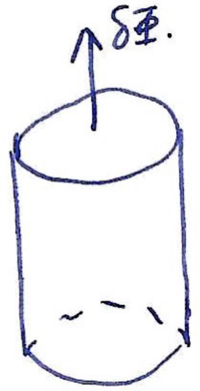


$$\varepsilon = \varepsilon_n(k_y)$$

n：朗道能级指标

k_y：晶格动量

额外加一个通量.



$$\delta\Phi = \iint B\cdot ds = \oint A\cdot dl$$
$$= A_y L_y$$
$$A_y = \frac{\delta\Phi}{L_y}$$

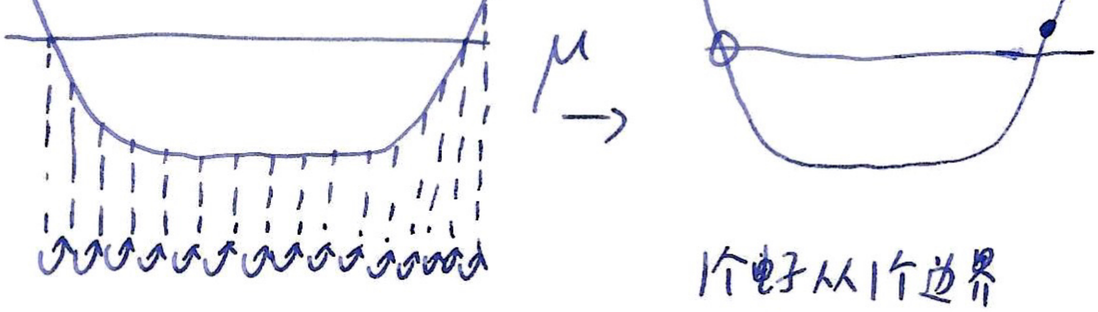
$$\varepsilon_n(k_y) \to \varepsilon_n\left(k_y + \frac{\delta\Phi}{L_y}\frac{e}{\hbar}\right)$$

取 $\delta\Phi = \frac{h}{e} = \Phi_0$

则 $\frac{\Phi_0}{L_y}\frac{e}{\hbar} = \frac{2\pi}{L_y}$

而周期边条件对 k_y 的限制亦为 $k_y = \frac{2\pi}{L_y}\cdot m,\ m=0,\pm1,\cdots$.

故. $\delta\Phi$ 使得 k_y 正好偏离1个单位.



1个电子从1个边界移至另1个边界.

1个朗道能级. $\delta E = \pm eE \cdot L_x$. , n个朗道能级与$\mu$相交. $\delta E = \pm n eEL_x$.

$$\langle J_y \rangle = -\frac{\delta E}{\delta A_y}$$

$$= -\frac{\pm eEL_x \cdot n}{\frac{h}{e}\frac{1}{L_y}}$$

$$= \mp n\frac{e^2}{h} L_x L_y \cdot E$$

单位面积的流

$$j_y = \frac{\langle J_y \rangle}{L_x L_y} = \mp n \frac{e^2}{h} E_x$$

$$\sigma_{yx} = \mp n \frac{e^2}{h}.$$

量子化！