

Universal Theory of Nonlinear Luttinger Liquids

Adilet Imambekov and Leonid I. Glazman*

One-dimensional quantum fluids are conventionally described by using an effective hydrodynamic approach known as Luttinger liquid theory. As the principal simplification, a generic spectrum of the constituent particles is replaced by a linear one, which leads to a linear hydrodynamic theory. We show that to describe the measurable dynamic response functions one needs to take into account the nonlinearity of the generic spectrum and thus of the resulting quantum hydrodynamic theory. This nonlinearity leads, for example, to a qualitative change in the behavior of the spectral function. The universal theory developed in this article is applicable to a wide class of one-dimensional fermionic, bosonic, and spin systems.

The development of the universal effective description of many-body phenomena is a central problem of the condensed matter theory. The hydrodynamic approach known as Luttinger liquid (LL) theory (1–3) is routinely applied to one-dimensional (1D) interacting systems. As a crucial simplification, a generic spectrum of the constituent particles is replaced by a linear one, leading to a linear hydrodynamic theory, which is nothing but a collection of non-interacting oscillators. However, to understand a variety of phenomena, such as Coulomb drag between quantum wires (4), momentum-resolved tunneling of electrons in nanowires (5), and neutron scattering off spin chains (3), one needs to take into account the nonlinearity of the spectrum. From classical physics, it is known that the existence of nonlinearities may result in qualitatively new phenomena, such as propagation of solitons and appearance of shock waves. These phenomena take place in a variety of experimental situations because classical nonlinear hydrodynamics is universal: It is phenomenologically derived from simple assumptions, which do not rely on microscopic details. Although description of linear quantum hydrodynamic theory requires only quantum mechanical treatment of non-interacting oscillators, formulation of nonlinear quantum hydrodynamics remains a challenging task because of divergences typical of nonlinear quantum field theories. In this article, we develop a universal theory of 1D quantum liquids that includes nonlinear hydrodynamic effects, leading to qualitative changes in predictions for dynamic response functions (e.g., spectral function).

If the 1D quantum many-body problem for fermions is simplified by replacing a generic spectrum of particles by a linear one [the Tomonaga-Luttinger (TL) model (6–8)], it becomes solvable at any interaction strength. The Lorentz invariance

introduced by this simplification protects the existence of well-defined elementary excitations with linear dispersion relation. These excitations are quantized waves of density propagating with a velocity v . Adding a fermion to a 1D system described by the TL model requires creation of multiple elementary excitations (9, 10). This can be seen from the form of the fermionic single-particle spectral function $A(p, \omega)$, which describes the probability of tunneling a fermion with given momentum p and energy ω into the system [see Supporting Online Material (SOM) for the precise definition (11)]. The spectral function has a power-

law singularity at the energy of collective excitation $\omega = vp$ (see eq. S4). The corresponding exponent is determined only by the universal LL parameter K [the latter is expressed in terms of the density, compressibility, and sound velocity v , the three low-energy properties of 1D liquid (3)].

In the phenomenological LL approach, energy scale $p^2/(2m^*)$ is fully dispensed with (the effective mass m^* characterizes spectrum nonlinearity at $p = 0$ and is defined below). The conventional justification for such simplification is irrelevance, in the renormalization group sense, of the nonlinearity (2). Indeed, the irrelevant terms hardly affect the fermion propagator away from the singular lines in space-time, $x \pm vt \gg \sqrt{t/m^*}$. However, it is the vicinity of these lines that defines the nature of singular behavior of the spectral function. We show here that for all spinless 1D fermionic models with short range interactions the single-particle spectral function at $p \ll k_f$ is universal. In the vicinity of Fermi wave vector $+k_f$ and $p, \omega > 0$, for example, $A(p, \omega)$ is a universal function of a single argument

$$A(p, \omega) \propto A(\epsilon), \quad \epsilon = \frac{\omega - vp}{p^2/2m_*} \quad (1)$$

(hereinafter p is measured from the closest Fermi point, and we use units with $\hbar = 1$). The new nontrivial function $A(\epsilon)$ is very different from the LL theory predictions, yet it depends only on the LL parameter K . The asymptote of $A(\epsilon)$ at $\epsilon \gg$

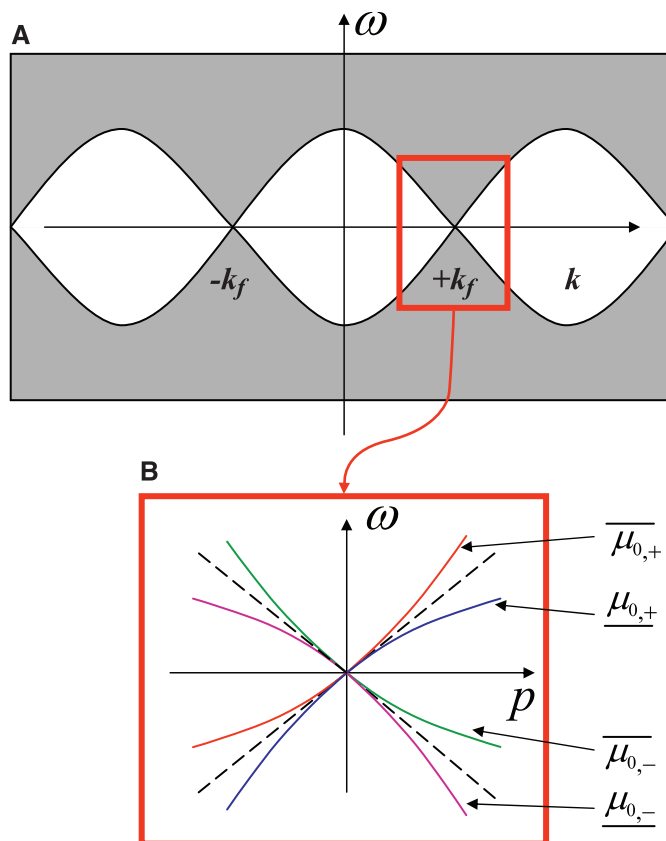


Fig. 1. Spectral function. **(A)** Spectral function $A(k, \omega)$ in momentum-energy plane. Shaded areas indicate the regions where $A(k, \omega) \neq 0$. The region with $\omega > 0$ corresponds to the particle part of the spectrum, and the region with $\omega < 0$ corresponds to the hole part of the spectrum. **(B)** Close-up view of the vicinity of $k \approx +k_f$, where $p = k - k_f$. Notations of μ indicate which exponents presented in Table 1 should be used in Eq. 6. Notations for exponents near $k \approx (2n + 1)k_f$ are obtained by substituting corresponding n instead of $n = 0$.

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1 does reproduce the LL theory predictions, eq. S4, but at $|\varepsilon \pm 1| \ll 1$ the spectral function is described by power-law asymptotes with new exponents. The exponents are different from the predictions of the LL theory but can still be analytically expressed in terms of K . We find numerically the universal single-variable crossover function $A(\varepsilon)$, by relating it to the nonlinear dynamics in nonequilibrium Fermi gases (12–15). We also briefly discuss applications of our results for bosonic and spin systems.

The spectral function could be measured in tunneling experiments with electrons in nanowires (5) and cold atoms in elongated traps (16, 17). A closely related object, transverse dynamic spin structure factor, is measurable by neutron scattering off 1D spin liquids placed in a magnetic field (3). The universal crossover function and its analytically obtained asymptotes also provide one with a test for numerical methods to evaluate many-body dynamics of 1D models, for example, using density-matrix renormalization group algorithms (18, 19).

Within a LL approach, fermionic field Ψ is expanded by using its components near Fermi points as $\Psi(x,t) \approx \Psi_R(x,t)e^{ik_F x} + \Psi_L(x,t)e^{-ik_F x}$, and the kinetic energy term in the Hamiltonian is linearized. Solution of the linearized model can be described by using free bosonic fields with linear dispersion. Fermionic operators are expressed as exponentials of free bosonic fields, and their correlations are easily evaluated. Including the nonlinearity of the spectrum of constituent fermions leads to interactions between bosonic fields (2, 20). One cannot treat such interactions perturbatively in bosonic language in the vicinity of the line $\omega = v p$ because even in the second order of perturbation theory corrections diverge there (21). Physically this happens because conservation laws of energy and momentum are satisfied simultaneously for waves with linear dispersion. Thus, two wave packets spend an infinite amount of time near each other, leading to an ill-defined perturbation theory. To understand the effects of nonlinear

spectrum, it is more convenient to work in the fermionic representation. Recently a connection between dynamic response functions of 1D quantum liquids and well-known Fermi edge singularity was elucidated (22, 23). It allowed one to evaluate dynamic structure factor $S(p,\omega)$ and spectral function perturbatively in the interaction between fermions. Moreover, it established the form of the effective Hamiltonian defining the true low-energy behavior of a liquid composed of generic particles with nonlinear dispersion relation. For some integrable 1D models, it is possible to determine the parameters of the effective Hamiltonian nonperturbatively by means of Bethe ansatz (24–26).

The Hamiltonian of the TL model may be recast into the Hamiltonian of free fermionic quasiparticles (27–29) having a linear spectrum:

$$\tilde{H}_1 = iv \int dx [\tilde{\Psi}_L^\dagger(x) \nabla \tilde{\Psi}_L(x) - : \tilde{\Psi}_R^\dagger(x) \nabla \tilde{\Psi}_R(x) :] \quad (2)$$

Here $\tilde{\Psi}_{R(L)}^\dagger(x)$ and $\tilde{\Psi}_{R(L)}(x)$ are creation and annihilation operators for quasiparticles on the right (left) branch, satisfying usual fermionic commutation relations. Colons indicate the normal ordering with respect to filled Fermi seas: for right (left) branch all states with negative (positive) momenta are occupied. The density of quasiparticles $\tilde{\rho}_{R(L)}(x) = : \tilde{\Psi}_{R(L)}^\dagger(x) \tilde{\Psi}_{R(L)}(x) :$ is simply related to the density of fermions in the TL model $\rho_{R(L)}(x) = \Psi_{R(L)}^\dagger(x) \Psi_{R(L)}(x)$. Because the canonical transformation that diagonalizes the TL Hamiltonian is a Bogoliubov rotation in the space of particle-hole excitations, such a relation is linear, $\rho_R(x) + \rho_L(x) = K[\tilde{\rho}_R(x) + \tilde{\rho}_L(x)]$. Fermionic operators are related to fermionic quasiparticles using “string” operators $\tilde{F}_{R(L)}^\dagger(x)$ as (e.g., for right-movers)

$$\Psi_R^\dagger(x) = \tilde{F}_R^\dagger(x) \tilde{\Psi}_R^\dagger(x), \tilde{F}_R^\dagger(x) = \exp \left\{ i \int^x dy [\delta_+ \tilde{\rho}_R(y) + \delta_- \tilde{\rho}_L(y)] \right\} \quad (3)$$

Here we have introduced parameters

$$\frac{\delta_+}{2\pi} = 1 - \frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2} < 0, \quad \frac{\delta_-}{2\pi} = \frac{1}{2\sqrt{K}} - \frac{\sqrt{K}}{2} \quad (4)$$

Using Eqs. 3 and 4 together with Eq. 2, one can obtain the usual results for Green's function of the TL model (27).

If one wants to consider effects of nonlinearity, one has to include terms that are less relevant in the renormalization group sense into quasiparticle Hamiltonian. One such term is the nonlinearity of the spectrum of quasiparticles:

$$\tilde{H}_2 = \frac{1}{2m_*} \int dx [: (\nabla \tilde{\Psi}_L^\dagger)(\nabla \tilde{\Psi}_L) : + : (\nabla \tilde{\Psi}_R^\dagger)(\nabla \tilde{\Psi}_R) :] \quad (5)$$

Here m_* is the effective mass, which can be related (20) to low-energy properties as $1/m_* = v/K^{1/2} \partial v / \partial h + v^2 / (2K^{3/2}) \partial K / \partial h$, where h is the chemical potential.

In principle, there is another term that needs to be included together with Eq. 5: It amounts to interaction between quasiparticles created by operators $\tilde{\Psi}_{L,R}^\dagger$. It can be shown (28), however, that in the limit of small p interactions between quasiparticles are weak and can be treated perturbatively, along the lines of (22, 23). Perturbation theory is valid as long as the interaction between the original fermions (created by Ψ^\dagger) is short-ranged. Interactions between quasiparticles are responsible for weak singularities in $S(p,\omega)$ near $\omega = v p \pm p^2 / (2m_*)$, large- ω tails of $S(p,\omega)$ (22), and for possible finite $\propto p^8$ smearing (23) of some of the singularities of $A(p,\omega)$. All these effects vanish as long as one is interested in the scaling limit $p \rightarrow 0, \varepsilon \rightarrow \text{const}$; see SOM (II) for more detailed discussion. For models with interactions decaying as $\propto 1/x^2$ or slower, nonanalytic dependence of interactions on momentum be-

Table 1. Universal exponents for spectral function. Notations are indicated in Fig. 1, and parameters δ_\pm defined by Eq. 4 are functions of K only. Note that $\mu_{n,+} = \mu_{-n-1,-}$, which follows from the $k \rightarrow -k$ symmetry.

$\overline{\mu_{n,+}}$	$1 - \frac{1}{2} \left[2n - (2n+1) \frac{\delta_+ + \delta_-}{2\pi} \right]^2 - \frac{1}{2} \left(\frac{\delta_+ - \delta_-}{2\pi} \right)^2$
$\underline{\mu_{n,+}}$	$1 - \frac{1}{2} \left[2n + 2 - (2n+1) \frac{\delta_+ + \delta_-}{2\pi} \right]^2 - \frac{1}{2} \left(2 - \frac{\delta_+ - \delta_-}{2\pi} \right)^2$
$\overline{\mu_{n,-}}$	$1 - \frac{1}{2} \left[2n + 2 - (2n+1) \frac{\delta_+ + \delta_-}{2\pi} \right]^2 - \frac{1}{2} \left(\frac{\delta_+ - \delta_-}{2\pi} \right)^2$
$\underline{\mu_{n,-}}$	$1 - \frac{1}{2} \left[2n - (2n+1) \frac{\delta_+ + \delta_-}{2\pi} \right]^2 - \frac{1}{2} \left(2 - \frac{\delta_+ - \delta_-}{2\pi} \right)^2$

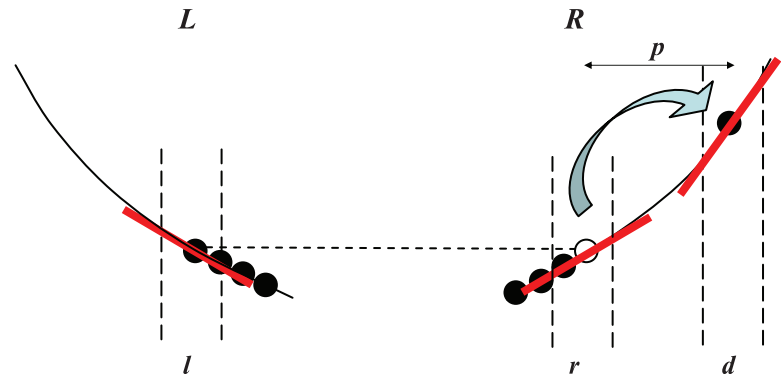


Fig. 2. Reduction to the effective Hamiltonian. We show excitations contributing to the singularity at $|\omega - (vp + \frac{p^2}{2m_*})| \ll \frac{p^2}{2m_*}$. The Hamiltonian given by Eqs. 2 and 5 is reduced to the three-subband model in Eqs. 7 and 8.

comes possible, and one cannot neglect interactions between quasiparticles. This can be already seen from perturbative calculations (23).

The spectral function $A(p, \omega)$ gets modified by the spectrum nonlinearity in a profound way because the dynamics of the string operators $\tilde{F}_{R(L)}^\dagger(x, t)$ in Eq. 3 becomes nonlinear. Effective mass m_* defines the energy scale $\sim p^2/(2m_*)$ near $\omega = v p$ where modifications from the TL model take place. Because parameters δ_\pm defining $\tilde{F}_{R(L)}^\dagger(x, t)$ are universally related to K , full form of the crossover written in terms of a variable ε is a universal function of K . Investigation of the properties of crossover function $A(\varepsilon)$ is the main subject of the present article.

Before proceeding to discuss the form of the universal crossover, let us consider the main new features of $A(p, \omega)$ that arise because of nonlinear spectrum. We find that in the vicinity of each low-energy region $k \approx (2n + 1)k_f$ spectral function $A(p, \omega)$ has a power-law behavior near frequencies $\pm[vp \pm p^2/(2m_*)]$, which is related to orthogonality catastrophe phenomenon (22, 23):

$$A(p, \omega) \propto \text{const} + \left| \frac{1}{\omega \pm \left(vp \pm \frac{p^2}{2m_*} \right)} \right|^\mu \quad (6)$$

and notations for μ are shown in Fig. 1. Such power-law behavior results from multiple low-energy particle-hole excitations near left and right Fermi points, which are created when “high energy” fermion tunnels into the system.

To be specific, let us focus on the vicinity of $+k_f$ for $p > 0$ and $\omega > 0$. Because the fermion that tunnels into the system has a momentum near $+k_f$ and energy of the system increases for $\omega > 0$, we need to consider only the correlator $\langle \Psi_R(x, t) \Psi_R^\dagger(0, 0) \rangle$.

Let us first discuss the exponent $\overline{\mu}_{0,+}$ at the edge $\left| \omega - \left(vp + \frac{p^2}{2m_*} \right) \right| \ll \frac{p^2}{2m_*}$. To understand

its origin, one has to understand the states that can be created by Ψ_R^\dagger , when the energy of the tunneling fermion is in the vicinity of the edge. From energy and momentum conservation, such state is given by a single fermionic quasiparticle with “large” momentum $\approx p$ and multiple low-energy particle-hole excitations with momenta much smaller than p , as indicated in Fig. 2. Then one can neglect all other states (22, 23) and project quasiparticle operators $\tilde{\Psi}_R(x)$ and $\tilde{\Psi}_L(x)$ onto narrow (of the width much smaller than p) subbands r, d , and l as $\tilde{\Psi}_R(x) \approx \tilde{\Psi}_r(x) + e^{ipx} \tilde{d}(x), \tilde{\Psi}_L(x) \approx \tilde{\Psi}_l(x)$.

The effective Hamiltonian determining the evolution of these states is obtained by projecting $\tilde{H}_1 + \tilde{H}_2$ onto subbands r, l , and d and linearizing the corresponding spectra:

$$\tilde{H}_{r,l} = i v \int dx [\tilde{\Psi}_l^\dagger(x) \nabla \tilde{\Psi}_l(x) : - : \tilde{\Psi}_r^\dagger(x) \nabla \tilde{\Psi}_r(x) :] \quad (7)$$

$$\tilde{H}_d = \int dx \tilde{d}^\dagger(x) \left[v p + \frac{p^2}{2m_*} - i \left(v + \frac{p}{m_*} \right) \nabla \right] \tilde{d}(x) \quad (8)$$

The Green's function factorizes as $\propto e^{ipx} \langle \tilde{d}(x, t) \tilde{d}^\dagger(0, 0) \rangle_{\tilde{H}_d} \langle \tilde{F}_r(x, t) \tilde{F}_r^\dagger(0, 0) \rangle_{\tilde{H}_{r,l}}$. To obtain string operators $\tilde{F}_r, \tilde{F}_r^\dagger$ from Eq. 3, one should keep only r and l components of the density there. The free-particle correlator $\langle \tilde{d}(x, t) \tilde{d}^\dagger(0, 0) \rangle_{\tilde{H}_d}$ equals $\propto e^{-i \left(vp + \frac{p^2}{2m_*} \right) t} \delta \left[x - \left(v + \frac{p}{m_*} \right) t \right]$, and string correlator can be bosonized and evaluated (3) in a usual way as $\langle \tilde{F}_r(x, t) \tilde{F}_r^\dagger(0, 0) \rangle_{\tilde{H}_{r,l}} \Big|_{x=(v+\frac{p}{m_*})t} \propto t^{-[\delta_-/(2\pi)]^2 - [\delta_+/(2\pi)]^2}$. Taking Fourier transform of $\langle \Psi_R(x, t) \Psi_R^\dagger(0, 0) \rangle$, we obtain the universal exponent

$$\overline{\mu}_{0,+} = 1 - \left(\frac{\delta_-}{2\pi} \right)^2 - \left(\frac{\delta_+}{2\pi} \right)^2 \quad (9)$$

Analogously, exponent $\underline{\mu}_{0,+}$ for $\omega - \left(vp - \frac{p^2}{2m_*} \right) \ll$

$\frac{p^2}{2m_*}$ is determined by configurations with one quasihole with the momentum $\approx -p$, two quasiparticles near right Fermi point, and low-energy particle-hole excitations. One can again reduce the problem to three-subband model and bosonize states near right and left Fermi points. This way, one obtains the exponent

$$\underline{\mu}_{0,+} = 1 - \left(\frac{\delta_-}{2\pi} \right)^2 - \left(2 - \frac{\delta_+}{2\pi} \right)^2 < -3 \quad (10)$$

New exponents given by Eqs. 9 and 10 are clearly different from the result for the TL model in eq. S4, which corresponds to the exponent $1 - [\delta_-/(2\pi)]^2$.

Configurations responsible for the remaining exponents $\overline{\mu}_{0,-}, \underline{\mu}_{0,-}$ consist of “high energy” particle-hole excitation on the left branch, particle at the right Fermi point, and low-energy excitations on left and right branches. Singularities near $k \approx (2n + 1)k_f$ also include n low-energy particle-hole pairs with momentum $\approx 2nk_f$. All exponents can be obtained by using projections onto three-subband models, and the results are summarized in Table 1.

We now discuss the results for the universal crossover function $A(\varepsilon)$ in the vicinity of $+k_f$ for $p, \omega > 0$ [details of the derivations are available in SOM (11)]. The answer is defined by a universal function $D(y)$, determined only by δ_+ and normalized as $\int_{-1}^1 D(y) dy = 1$. By using $D(y)$,

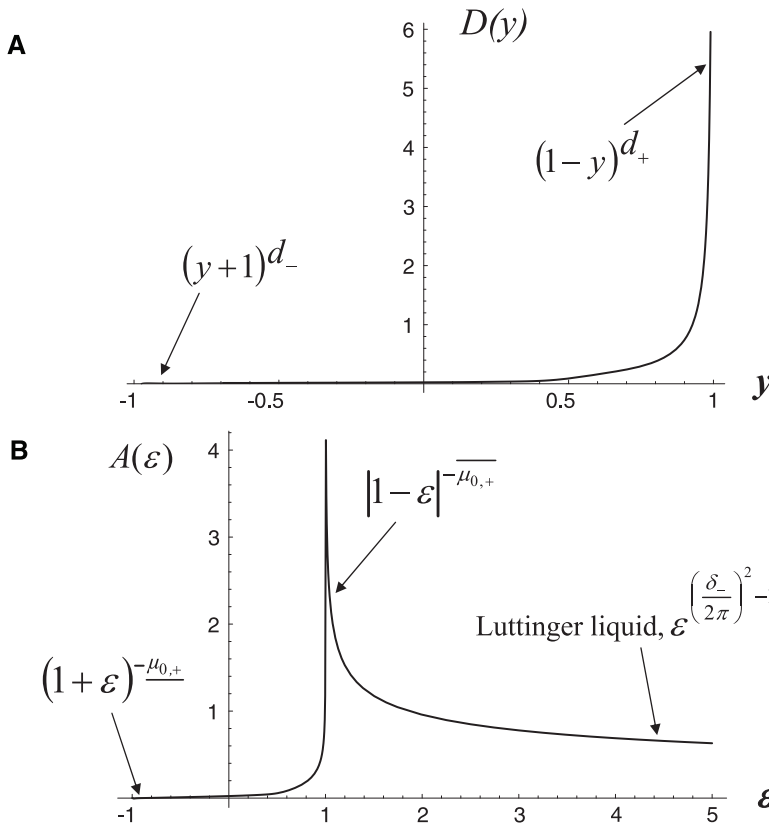


Fig. 3. Universal crossover. **(A)** Universal crossover function $D(y)$ for $K = 4.54$ and the corresponding values $\delta_+/(2\pi) = -0.3$ and $\delta_-/(2\pi) = -0.83$; see Eq. 4. Exponents d_\pm defining the asymptotic behavior at $y \rightarrow \pm 1$ are given by Eq. 12. **(B)** Universal function $A(\varepsilon)$ for $K = 4.54$. Exponents $\overline{\mu}_{0,+}$ and $\underline{\mu}_{0,+}$ defining the asymptotic behavior at $\varepsilon \rightarrow \pm 1$ are given by Eqs. 9 and 10. The ratio of prefactors determining the asymmetry of the singularity at $\varepsilon = 1$, see Eq. 13, equals 2.96 for $K = 4.54$.

spectral function can be written as a convolution of contributions from the left and right branches. Universal function $A(\epsilon)$ in Eq. 1 is related to $D(y)$ as

$$A(\epsilon) = \int_{-1}^1 dy D(y) \theta(\epsilon - y) (\epsilon - y)^{\frac{\delta_{\pm}}{2\pi} - 1} \quad (11)$$

One can analytically obtain limiting behavior of $D(y)$ for $y \rightarrow \pm 1$ from Eqs. 9 to 11 as $D(y) \propto (1 \mp y)^{d_{\pm}}$ for $y \rightarrow \pm 1$, where

$$d_{+} = \left(\frac{\delta_{+}}{2\pi}\right)^2 - 1, \quad d_{-} = \left(2 - \frac{\delta_{+}}{2\pi}\right)^2 - 1 > 3 \quad (12)$$

At moderate interaction strength, $\overline{\mu_{0,+}} > 0$, function $A(\epsilon)$ diverges at $\epsilon = 1$. Then the ratio of the prefactors above and below the singular line is universal,

$$\lim_{|\delta\epsilon| \rightarrow 0} \frac{A(1 + |\delta\epsilon|)}{A(1 - |\delta\epsilon|)} = \frac{\Gamma\left[\frac{\delta_{+}}{2\pi}\right]^2 \Gamma\left[1 - \left(\frac{\delta_{+}}{2\pi}\right)^2\right]}{\Gamma\left[\frac{\delta_{-}}{2\pi}\right]^2 \Gamma\left[1 - \left(\frac{\delta_{-}}{2\pi}\right)^2\right]} \quad (13)$$

To evaluate $D(y)$ away from the edges, one should be able to calculate the dynamics of chiral vertex operators (11). For a nonlinear spectrum, this is a very nontrivial problem, the analytic solution of which is not known. Similar correlators have attracted attention recently (12, 13), and their connection to the nonlinear quantum shock wave dynamics and nonlinear differential equations has been discussed. Although it might be possible to proceed similarly for the evaluation of $D(y)$, it is not clear whether nonlinear differential equations obtained this way will have an analytic solution. We use an alternative approach of (14, 15), which allows us to develop a representation of $D(y)$ in terms of certain determinants built of single-particle (rather than many-body) states. These determinants can be evaluated numerically, which practically solves the problem of finding $D(y)$. Representative results for $D(y)$ and $A(\epsilon)$ for $K = 4.54$ are shown in Fig. 3.

The universal Hamiltonian given by Eqs. 2 and 5 can be also used to describe gapless bosonic and spin $-\frac{1}{2}$ systems away from particle hole symmetric ground states. We present main results on singularities of their dynamic response functions in SOM (11).

We have constructed universal low-energy theory of a wide class of interacting 1D quantum liquids without resorting to the simplifications of the Tomonaga-Luttinger model accepted in the phenomenological LL description. Unlike the latter, we keep the nonlinear dispersion relation of the fermions intact. The replacement of the dispersion relation by a linear one, $\omega = vp$, results in an artificial introduction of Lorentz invariance into the system. Although not affecting the low-energy behavior of local properties (such as the local tunneling density of states), the introduced

symmetry alters qualitatively the predictions for the momentum-resolved quantities, such as the spectral function. Keeping the nonlinearity allows us to find the generic low-energy behavior of the dynamic response functions of a system of interacting fermions, bosons, and spins. Possible extensions of our theory should be able to describe the effects of finite temperature, spin systems at particle-hole symmetric points, systems with long-range interactions, and fermions with spin.

References and Notes

1. F. D. M. Haldane, *Phys. Rev. Lett.* **47**, 1840 (1981).
2. F. D. M. Haldane, *J. Phys. C* **14**, 2585 (1981).
3. T. Giamarchi, *Quantum Physics in One Dimension* (Oxford Univ. Press, New York, 2004).
4. M. Pustilnik, E. G. Mishchenko, L. I. Glazman, A. V. Andreev, *Phys. Rev. Lett.* **91**, 126805 (2003).
5. O. M. Auslaender *et al.*, *Science* **295**, 825 (2002).
6. S. Tomonaga, *Prog. Theor. Phys.* **5**, 544 (1950).
7. J. M. Luttinger, *J. Math. Phys.* **4**, 1154 (1963).
8. D. C. Mattis, E. H. Lieb, *J. Math. Phys.* **6**, 304 (1965).
9. I. E. Dzyaloshinskii, A. I. Larkin, *Sov. Phys. JETP* **38**, 202 (1974).
10. A. Luther, I. Peschel, *Phys. Rev. B* **9**, 2911 (1974).
11. Materials and methods are available as supporting material on Science Online.
12. E. Bettelheim, A. G. Abanov, P. Wiegmann, *Phys. Rev. Lett.* **97**, 246402 (2006).
13. E. Bettelheim, A. G. Abanov, P. Wiegmann, *J. Phys. A* **41**, 392003 (2008).
14. D. A. Abanin, L. S. Levitov, *Phys. Rev. Lett.* **93**, 126802 (2004).
15. D. A. Abanin, L. S. Levitov, *Phys. Rev. Lett.* **94**, 186803 (2005).
16. L.-M. Duan, *Phys. Rev. Lett.* **96**, 103201 (2006).
17. T.-L. Dao, A. Georges, J. Dalibard, C. Salomon, I. Carusotto, *Phys. Rev. Lett.* **98**, 240402 (2007).
18. S. R. White, *Phys. Rev. Lett.* **69**, 2863 (1992).
19. U. Schollwöck, *Rev. Mod. Phys.* **77**, 259 (2005).
20. R. G. Pereira *et al.*, *Phys. Rev. Lett.* **96**, 257202 (2006).
21. K. Samokhin, *J. Phys.* **10**, L533 (1998).
22. M. Pustilnik, M. Khodas, A. Kamenev, L. I. Glazman, *Phys. Rev. Lett.* **96**, 196405 (2006).
23. M. Khodas, M. Pustilnik, A. Kamenev, L. I. Glazman, *Phys. Rev. B* **76**, 155402 (2007).
24. R. G. Pereira, S. R. White, I. Affleck, *Phys. Rev. Lett.* **100**, 027206 (2008).
25. V. V. Cheianov, M. Pustilnik, *Phys. Rev. Lett.* **100**, 126403 (2008).
26. A. Imambekov, L. I. Glazman, *Phys. Rev. Lett.* **100**, 206805 (2008).
27. A. V. Rozhkov, *Eur. Phys. J. B* **47**, 193 (2005).
28. A. V. Rozhkov, *Phys. Rev. B* **74**, 245123 (2006).
29. A. V. Rozhkov, *Phys. Rev. B* **77**, 125109 (2008).
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Supporting Online Material

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Materials and Methods

Fig. S1

Table S1

References

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Direct Measurement of Molecular Mobility in Actively Deformed Polymer Glasses

Hau-Nan Lee, Keewook Paeng, Stephen F. Swallen, M. D. Ediger

When sufficient force is applied to a glassy polymer, it begins to deform through movement of the polymer chains. We used an optical photobleaching technique to quantitatively measure changes in molecular mobility during the active deformation of a polymer glass [poly(methyl methacrylate)]. Segmental mobility increases by up to a factor of 1000 during uniaxial tensile creep. Although the Eyring model can describe the increase in mobility at low stress, it fails to describe mobility after flow onset. In this regime, mobility is strongly accelerated and the distribution of relaxation times narrows substantially, indicating a more homogeneous ensemble of local environments. At even larger stresses, in the strain-hardening regime, mobility decreases with increasing stress. Consistent with the view that stress-induced mobility allows plastic flow in polymer glasses, we observed a strong correlation between strain rate and segmental mobility during creep.

Glasses form when molecular motion becomes slow, and thus liquidlike flow in a glass would seem impossible by definition. Nevertheless, polymer glasses under stress can yield and undergo plastic flow (1). In this process, the glass dissipates enormous amounts of energy without breaking. This toughness is

the critical design requirement in many applications, and efforts to understand it go back more than 70 years. In 1936, Eyring (2) proposed a model in which external loading lowers the energy barriers for molecular motion and thus effectively transforms a glass into a viscous liquid. Other workers (1, 3–7) have modified Eyring's approach in important ways while maintaining the central idea that stress can induce molecular mobility.

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Supporting Online Material for
Universal Theory of Nonlinear Luttinger Liquids
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Supporting Online Material: Universal Theory of Nonlinear Luttinger Liquids

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Materials and Methods 1: Definitions of the dynamic correlation functions

We are interested mostly in the zero-temperature spectral function

$$A(k, \omega) = -\frac{1}{\pi} \text{Im} G^{\text{ret}}(k, \omega), \quad (\text{S1})$$

where retarded Green's function $G^{\text{ret}}(k, \omega)$ is defined by (S1)

$$G^{\text{ret}}(k, \omega) = -i \int \int dx dt e^{i(\omega t - kx)} \times \langle \Psi(x, t) \Psi^\dagger(0, 0) + \Psi^\dagger(0, 0) \Psi(x, t) \rangle \theta(t), \quad (\text{S2})$$

and dynamic structure factor (DSF)

$$S(p, \omega) = \int dx dt e^{i(\omega t - px)} \langle \rho(x, t) \rho(0, 0) \rangle. \quad (\text{S3})$$

Here $\Psi^\dagger(x, t)$, $\Psi(x, t)$ and $\rho(x, t)$ are fermionic or bosonic creation, annihilation and density operators, respectively. Energy ω is measured relative to the chemical potential, so $A(k, \omega)$ for $\omega > 0$ ($\omega < 0$) describes the response of the system to an addition of an extra particle (hole).

Materials and Methods 2: Universal crossover

In this section we present the details of the derivations the universal crossover function $A(\varepsilon)$ in the vicinity of $+k_f$ for $p, \omega > 0$. Before proceeding to the case of the nonlinear spectrum, let us present the derivation of a conventional result for the Tomonaga-Luttinger model,

$$A(p, \omega) \propto (\omega - vp)^{\frac{1}{4}(K + \frac{1}{K} - 2) - 1} \theta(\omega - vp), \quad (\text{S4})$$

which allows for an easy generalization to the nonlinear case.

Retarded Green's function for fermions near $+k_f$ can be written (S2) as a product of two terms, determined by left and right Fermi points. Due to linear spectrum, they depend on combinations $vt + x$ and $vt - x$, respectively:

$$G_{\text{R}}^{\text{ret}}(x, t) \propto -i \frac{\theta(vt + x)}{(i(vt + x) + 0)^{(\frac{\delta_-}{2\pi})^2}} \frac{\theta(vt - x)}{(i(vt - x) + 0)^{(\frac{\delta_-}{2\pi})^2 + 1}}. \quad (\text{S5})$$

If one defines $L(x, t)$ and $R(x, t)$ as

$$L(x, t) \propto \frac{1}{(i(vt + x) + 0)^{(\frac{\delta_-}{2\pi})^2}}, \quad R(x, t) \propto \frac{1}{(i(vt - x) + 0)^{(\frac{\delta_-}{2\pi})^2 + 1}}, \quad (\text{S6})$$

then imaginary part of Fourier transform of $G_{\text{R}}^{\text{ret}}(x, t)$ can be represented as a convolution of two Fourier transforms of $L(x, t)$ and $R(x, t)$:

$$\text{Im}G_{\text{R}}^{\text{ret}}(p, \omega) = -i \int \frac{d\tilde{p}}{2\pi} \frac{d\tilde{\omega}}{2\pi} L(p - \tilde{p}, \omega - \tilde{\omega}) R(\tilde{p}, \tilde{\omega}), \quad (\text{S7})$$

where real positive functions $R(p, \omega)$ and $L(p, \omega)$ are equal to (up to a positive cut-off dependent prefactor)

$$L(p, \omega) \propto \delta(\omega + vp) \theta(\omega - vp) (\omega - vp)^{(\frac{\delta_-}{2\pi})^2 - 1}, \quad (\text{S8})$$

$$R(p, \omega) \propto \delta(\omega - vp) \theta(\omega + vp) (\omega + vp)^{(\frac{\delta_-}{2\pi})^2}. \quad (\text{S9})$$

Using Eq. S1 spectral function $A(p, \omega)$ can be written as

$$A(p, \omega) = \frac{1}{\pi} \int \frac{d\tilde{p}}{2\pi} \frac{d\tilde{\omega}}{2\pi} L(p - \tilde{p}, \omega - \tilde{\omega}) R(\tilde{p}, \tilde{\omega}). \quad (\text{S10})$$

Physically, $A(p, \omega)$ describes the probability of tunneling of a fermion with total energy ω and momentum p . Eqs. S8-S9 then mean that excitations which are created on the right (left) branch should lie on the respective mass shell and have positive (negative) momenta. From energy and momentum conservation laws, nonzero contribution to $A(p, \omega)$ for $\omega > vp$ comes only from a single point in the integral in Eq. S10, which correspond to the following arguments of functions $R(p_{\text{R}}, \omega_{\text{R}})$ and $L(p_{\text{L}}, \omega_{\text{L}})$ in the integrand:

$$\omega_{\text{R}} = vp_{\text{R}} = \frac{(\omega + vp)}{2}, \quad (\text{S11})$$

$$\omega_{\text{L}} = -vp_{\text{L}} = \frac{(\omega - vp)}{2}. \quad (\text{S12})$$

Since only $L(p_{\text{L}}, \omega_{\text{L}})$ is singular for $\omega \rightarrow vp$, for the Tomonaga-Luttinger model only the contribution due to the shake-up of low-energy excitations near the left Fermi point controls the

exponents at $\omega \approx vp$. For nonlinear spectrum, shake-up contributions from both left and right Fermi points determine the exponents, see e.g. Eq. 9.

For nonlinear spectrum, $G_R^{\text{ret}}(x, t)$ can still be represented as a product of two terms determined by left and right Fermi points. The primary modification which takes place is that delta-functions in Eqs. S8-S9 get broadened. Indeed, for nonlinear spectrum the total momentum of several left (right)-moving quasiparticles doesn't completely define their total energy, and the latter is allowed to vary up to $\pm p_{L(R)}^2/2m_*$. However, if one is interested in the scaling behavior of $A(p, \omega)$ for

$$\omega - vp = \varepsilon \frac{p^2}{2m_*}, \quad (\text{S13})$$

one can neglect the finite width of function L , since according to Eqs. S12 and S13 it is of the order $p_L^2 \propto p^4$, which vanishes in the scaling limit $p \rightarrow 0, \varepsilon \rightarrow \text{const}$. Broadening of function R , on the other hand, is important. As a result of it the momentum and energy on each branch can vary on the order $\sim p^2/(2m_*v)$ and $\sim p^2/(2m_*)$, respectively, around the values of Eqs. S11,S12.

To characterize the broadening of delta-function in Eq. S9, let us introduce a dimensionless positive function $D(y)$ defined by

$$R(p, vp + y \frac{p^2}{2m_*}) \propto D(y). \quad (\text{S14})$$

Since for total momentum p on the right branch $vp \pm p^2/(2m_*)$ is the highest (lowest) possible energy of a set of quasiparticles, $D(y) \neq 0$ only if $y \in (-1, 1)$. Universal function $D(y)$ is determined only by δ_+ , and we choose it to be normalized as

$$\int_{-1}^1 D(y) dy = 1. \quad (\text{S15})$$

We now discuss how to reduce the evaluation of $D(y)$ to a single-particle problem and solve it numerically. We use periodic boundary conditions on a circle of length L . Since from now on we will be dealing only with fermions at the right branch, we drop index R for clarity of notations, and set $m_* = 1/2$.

The chiral vertex correlation function which determines $D(y)$ can be written as

$$R'(x', t) = \langle e^{i\tilde{H}_2 t} \tilde{\Psi}(x') \exp \left[-i \int^{x'} dy \frac{\delta_+}{2\pi} \tilde{\rho}(y) \right] e^{-i\tilde{H}_2 t} \exp \left[i \int^0 dy \frac{\delta_+}{2\pi} \tilde{\rho}(y) \right] \tilde{\Psi}^\dagger(0) \rangle, \quad (\text{S16})$$

where one has to average over filled Fermi sea on the right branch. In Eq. S16 we took into account the effect of the linear-spectrum Hamiltonian \tilde{H}_1 , Eq. 2, by shifting $x' = x - vt$. Universal function $D(y)$ is determined by the Fourier transform of $R'(x', t)$ as

$$R'(p, t) = \int dx' e^{-ipx'} R'(x', t) \propto \int_{-1}^1 e^{ip^2 y t} D(y) dy. \quad (\text{S17})$$

In momentum space, $R'(x', t)$ can be written as

$$R'(x', t) = \prod_{k < 0} e^{ik^2 t} \sum_{p, p'} e^{ip'x'} \langle \tilde{\Psi}_{p'} e^{\mathcal{B}} e^{-i\mathcal{H}t} e^{\mathcal{A}} \tilde{\Psi}_p^\dagger \rangle, \quad (\text{S18})$$

where operators \mathcal{A} , \mathcal{B} and \mathcal{H} act in a many-body Hilbert space as

$$\mathcal{A} = -\frac{\delta_+}{2\pi} \sum_{p \neq p'} \frac{2\pi}{L(p-p')} \tilde{\Psi}_p^\dagger \tilde{\Psi}_{p'} = \sum_{p, p'} \hat{a}_{p, p'} \tilde{\Psi}_p^\dagger \tilde{\Psi}_{p'}, \quad (\text{S19})$$

$$\mathcal{B} = \frac{\delta_+}{2\pi} \sum_{p \neq p'} \frac{2\pi e^{-i(p-p')x}}{L(p-p')} \tilde{\Psi}_p^\dagger \tilde{\Psi}_{p'} = \sum_{p, p'} \hat{b}_{p, p'} \tilde{\Psi}_p^\dagger \tilde{\Psi}_{p'}, \quad (\text{S20})$$

$$\mathcal{H} = \sum_p p^2 \tilde{\Psi}_p^\dagger \tilde{\Psi}_p = \sum_{p, p'} \hat{h}_{p, p'} \tilde{\Psi}_p^\dagger \tilde{\Psi}_{p'}, \quad (\text{S21})$$

while \hat{a} , \hat{b} and \hat{h} are matrices acting in a single-particle Hilbert space. We introduce the density matrix

$$\hat{\rho} = \frac{1}{Z} e^{-\sum_p \lambda_p \tilde{\Psi}_p^\dagger \tilde{\Psi}_p}, e^{-\lambda_p} = \frac{n_p}{1 - n_p}, \quad (\text{S22})$$

where n_p is the occupation number of mode p , which we will be set to $\theta(-p)$ at the end of the calculation. Then Eq. S18 can be written as a trace over full many-body Hilbert space as

$$R'(x', t) = \prod_{k < 0} e^{ik^2 t} \sum_{p, p'} e^{ip'x'} \text{Tr} \left(\tilde{\Psi}_{p'} e^{\mathcal{B}} e^{-i\mathcal{H}t} e^{\mathcal{A}} \tilde{\Psi}_p^\dagger \hat{\rho} \right).$$

We use the relation

$$\tilde{\Psi}_p^\dagger \hat{\rho} = \hat{\rho} e^{\lambda_p} \tilde{\Psi}_p^\dagger, \quad (\text{S23})$$

which effectively restricts summation to $p > 0$, and the cyclic property of a trace to get

$$R'(x', t) = \prod_{k < 0} e^{ik^2 t} \sum_{p > 0, p'} e^{ip'x'} \text{Tr} \left(e^{\mathcal{B}} e^{-i\mathcal{H}t} e^{\mathcal{A}} \hat{\rho} \tilde{\Psi}_p^\dagger \tilde{\Psi}_{p'} \right).$$

This trace over many-body Hilbert space can be written via determinants of matrices acting in a single-particle Hilbert space as (S3-S7)

$$R'(x', t) = \prod_{k < 0} e^{ik^2 t} \sum_{p, p'} e^{ip'x'} (1 - n_p) \times \text{Det}(\hat{I} - \hat{n} + e^{\hat{b}} e^{-i\hat{h}t} e^{\hat{a}} \hat{n}) \left(\hat{n} + e^{-\hat{a}} e^{i\hat{h}t} e^{-\hat{b}} (1 - \hat{n}) \right)_{p', p}^{-1}, \quad (\text{S24})$$

where \hat{n} is a diagonal matrix with n_p on the diagonal.

To extract $D(y)$, one needs to evaluate $R'(p, t)$ defined by Eq. S17 at times

$$t_\alpha = \frac{\pi\alpha}{p^2} \quad (\text{S25})$$

for integer α . According to Eq. S17, it corresponds to Fourier series coefficient of $D(y)$:

$$R'(p, t_\alpha) \propto D_\alpha = \int_{-1}^1 e^{i\pi\alpha y} D(y) dy. \quad (\text{S26})$$

Function $D(y)$ can be written in terms of D_α as

$$D(y) = \frac{D_0}{2} + \sum_{\alpha=1}^{\infty} \text{Re} [D_\alpha e^{-i\pi\alpha y}] \quad (\text{S27})$$

Since for small enough δ_+ function $D(y)$ has a singularity given by Eq. 12 for $y \rightarrow 1$, one expects

$$D_\alpha \propto (-1)^\alpha \alpha^{-\left(\frac{\delta_+}{2\pi}\right)^2}, \text{ for } \alpha \rightarrow \infty. \quad (\text{S28})$$

We evaluate $R'(x', t_\alpha)$ for various x' using finite-dimensional Hilbert space of the size up to ~ 300 , and obtain its Fourier transform $R'(p, t_\alpha)$. Due to periodic boundary conditions and finite size effects, asymptote given by Eq. S28 doesn't hold for largest α obtained numerically. However, we find an excellent fit for sufficiently large α as

$$D_\alpha \propto e^{ic\alpha} (-1)^\alpha \left(\sin \frac{\alpha}{\tilde{\alpha}}\right)^{-\gamma}. \quad (\text{S29})$$

Exponent γ obtained using such fitting procedure equals $\left(\frac{\delta_+}{2\pi}\right)^2$ with a very high accuracy. In Eq. S29, $c \ll 1$ accounts for a possible shift of the frequency, while finite $\tilde{\alpha}$ accounts for finite size effects due to finite $kL/(2\pi)$ considered. We remove the latter effects by smoothly substituting D_α in Eq. S27 by

$$D_\alpha \propto e^{ia\alpha} (-1)^\alpha \left(\frac{\alpha}{\tilde{\alpha}}\right)^{-\gamma}, \quad (\text{S30})$$

for α larger than some intermediate $\alpha_* \ll \tilde{\alpha}$, and keeping numerical results for smaller α . The sum in Eq. S27 with D_α given by Eq. S30 can be written in terms of polylogarithmic functions. We sum the contributions to Eq. S27 coming from large α using polylogarithmic functions, while for smaller α we use numerical results.

The procedure to extract $D(y)$ described above is very robust, and is not sensitive to particular choice of parameters at the accuracy of about $\sim 2\%$ or 0.02, whichever is larger, for data presented in Fig. 3. As an independent check, it reproduces the result $D(-1) \approx 0$ very well. The correct value of the exponent $d_- > 3$ characterizing the asymptote $D(y \rightarrow -1)$ is harder to reproduce.

Materials and Methods 3: Bosonic and spin systems

The universal Hamiltonian given by Eqs. 2,5 can be also used to describe gapless bosonic and spin- $\frac{1}{2}$ systems away from particle-hole symmetric ground states. The only modification is the existence of an additional Jordan-Wigner "string" operator in the expression, in terms of fermions, for creation operator of bosons $\Psi_{\text{B}}^{\dagger}$ and for spin raising operator S^+ , respectively. Here we will only discuss the singularities.

For bosons, existence of new singularities in response functions of an integrable Lieb-Liniger (S8) model has been pointed out recently (S9) and in low-energy regime expressions for the exponents in terms of the Luttinger parameter K have been obtained. Same exponents can be obtained using the methods of the current article, which demonstrates their universality. Exponents in the vicinity of the low energy region $k \approx 2\pi n k_f$ are summarized in Table S1, and notations of bosonic exponents μ^b are indicated in Fig. S1.

For spin- $\frac{1}{2}$ systems, our results apply generally for the following antiferromagnetic ($J > 0$) Hamiltonian in a finite magnetic field h :

$$H = J \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \sum_{i>j} V_{i-j} S_i^z S_j^z - h \sum_i S_i^z.$$

Here $S_i^{x,y,z}$ are spin- $\frac{1}{2}$ operators, and V_i are assumed to decay faster than $1/i^2$, and to be small enough so that the system is gapless. We require finite magnetic field, since otherwise due to particle-hole symmetry one quite generally has $m_* = \infty$. In this case the regime discussed in present article disappears, as has been pointed out recently (S10) for an integrable XXZ model. While for fermionic systems without a lattice one expects $m_* > 0$, it is not necessarily the case for spins on a lattice. This can change the relative position of the singularities compared to Fig. 1. For small enough interactions one expects $m_* > 0$ ($m_* < 0$) for negative (positive) magnetic field h , although for small enough magnetic fields interactions can reverse the sign of m_* , see e.g. (S11).

We will be interested in transverse dynamic spin structure factor, defined by

$$S^{-+}(k, \omega) = \sum_j e^{-ikj} \int dt e^{i\omega t} \langle S_j^-(t) S_0^+(0) \rangle. \quad (\text{S31})$$

It is nonvanishing at low energies in the vicinity of $k = \pi$, as long as the spin chain remains gapless (S2,S12). Generalization of the approach described earlier leads to

$$S^{-+}(k, \omega) \propto \text{const} + \left| \frac{1}{\omega - \left(v|k - \pi| \pm \frac{(k-\pi)^2}{2m_*} \right)} \right|^{\pm \frac{1}{\sqrt{K}} - \frac{1}{2K}},$$

for $|k - \pi| \ll 1$. Here we have already expressed parameters δ_{\pm} as functions of K using Eq. 4.

Materials and Methods 4: Limits of applicability

In this section we discuss the limitations of and leading corrections to the universal results. One regime, when universal results are not applicable has been already pointed out above, and corresponds to $m_* = \infty$. Such situation generically arises for spin- $\frac{1}{2}$ system at half-filling, when leading correction to spectrum nonlinearity starts from terms $\propto p^3$. If leading $\propto p^2$ curvature of the spectrum is non-vanishing, then our results quite generically apply for

$$\frac{p}{k_f} \ll 1. \quad (\text{S32})$$

We show below that leading corrections to universal results are suppressed in powers of this small parameter. To be specific, we consider the modifications of singularities of fermionic $A(p, \omega)$ for $p > 0, \omega > 0$ in the vicinity of $+k_f$.

There are two types of terms which modify the universal Hamiltonian. One type of terms corresponds to higher order corrections to single-particle spectrum. Such terms merely shift the positions of the singularities, but do not change the exponents. Indeed, reduction to three-subband model only requires velocity of d -particle to be different from v . Since this happens already for leading spectrum nonlinearity $\propto p^2$, higher order curvature of the spectrum doesn't directly affect $\overline{\mu_{0,+}}$ and $\underline{\mu_{0,+}}$.

Second type of terms corresponds to irrelevant interactions between fermionic quasiparticles. One such term, which has the same scaling dimension as spectrum nonlinearity, is given by (S11,S13,S14)

$$\tilde{H}'_{\text{int}} = -i\tilde{g}' \int dx \left(\tilde{\rho}_R \left[: \tilde{\Psi}_L^\dagger \nabla \tilde{\Psi}_L : - : \nabla \tilde{\Psi}_L^\dagger \tilde{\Psi}_L : \right] - \tilde{\rho}_L \left[: \tilde{\Psi}_R^\dagger \nabla \tilde{\Psi}_R : - : \nabla \tilde{\Psi}_R^\dagger \tilde{\Psi}_R : \right] \right) \quad (\text{S33})$$

where g' can be related (S11,S14) to low energy properties similar to $1/m^*$, and generally these quantities are of the same order of magnitude. Effect of such interactions on e.g. $\overline{\mu_{0,+}}$ can be understood using the methods of Refs. (S15,S16). Indeed, after projection to three-subband model interactions lead to modification of phase shift δ_- of the order

$$\Delta\delta_- \sim \frac{g'p}{v_d - (-v)} \sim \frac{p}{k_f} \ll 1, \quad (\text{S34})$$

and thus lead to small corrections to $\overline{\mu_{0,+}}, \underline{\mu_{0,+}}$. Here v_d is the velocity of particle d , which equals

$$v_d = v + \frac{p}{m^*}. \quad (\text{S35})$$

Less relevant interactions between left and right branches lead to even stronger suppressed corrections to the exponents. One should note, that presence of finite g' also leads to $\propto p^8$ smearing of the singularity $\overline{\mu_{0,+}}$ for $\omega > 0$, while singularity $\underline{\mu_{0,+}}$, being a singularity at a true kinematic border, remains intact (S16).

Another irrelevant interaction term which modifies the exponents in linear order over p/k_f arises due to momentum dependence of interactions on the same branch,

$$\tilde{H}_{\text{int}}'' = \int dp V(p) (\tilde{\rho}_L(p)\tilde{\rho}_L(-p) + \tilde{\rho}_R(p)\tilde{\rho}_R(-p)). \quad (\text{S36})$$

Interaction $V(p)$ should vanish for $p \rightarrow 0$, and have the symmetry property $V(p) = V(-p)$. If one assumes that $V(p)$ is regular, then its expansion starts from the term $\propto p^2$, and correction to the phase shift δ_+ is of the order

$$\Delta\delta_+ \sim \frac{V(p)}{v_d - v} \sim \frac{p}{k_f} \ll 1. \quad (\text{S37})$$

However, for interactions that decay as or slower than $\propto 1/x^2$, momentum dependent part of $V(p)$ doesn't have to be regular. Indeed, for models with interactions decaying as $\propto 1/x^2$, one has $V(p) \sim |p|$, which leads to finite $\Delta\delta_+$ in the limit $p \rightarrow 0$, and a finite modification of the universal exponents. Thus our universal results do not apply to Haldane-Shastry (S17,S18) or Calogero-Sutherland (S19) models. In the case of the latter, this can be seen from explicit calculations (S16,S20).

Finally, we note that predictions of universal Hamiltonian for $S(p, \omega)$ for small p can be checked using sum rules. Universal Hamiltonian given by Eqs. 2,5 predicts (S21) that $S(p, \omega)$ at any interaction strength approaches the form characteristic for free fermions

$$S(p, \omega) = \frac{m_* K}{p} \theta\left(\frac{p^2}{2m_*} - |\omega - v|p\right), \quad (\text{S38})$$

once p becomes small enough. One can check, that for Galilean-invariant systems this result explicitly satisfies f-sum rule (S22)

$$\int \omega S(p, \omega) \frac{d\omega}{2\pi} = \frac{1}{2\pi} v K p^2 = \frac{np^2}{2m}. \quad (\text{S39})$$

In addition, for all systems compressibility sum rule (see e.g. Eq. 7.52 of Ref. (S22)) is also satisfied:

$$\lim_{p \rightarrow 0} \int \frac{1}{\omega} S(p, \omega) \frac{d\omega}{2\pi} = \frac{K}{2\pi v} = \frac{1}{2} \frac{\partial n}{\partial \mu}, \quad (\text{S40})$$

where in last equation we have used relation of K to compressibility, see e.g. Eq. 2.59 of Ref. (S2).

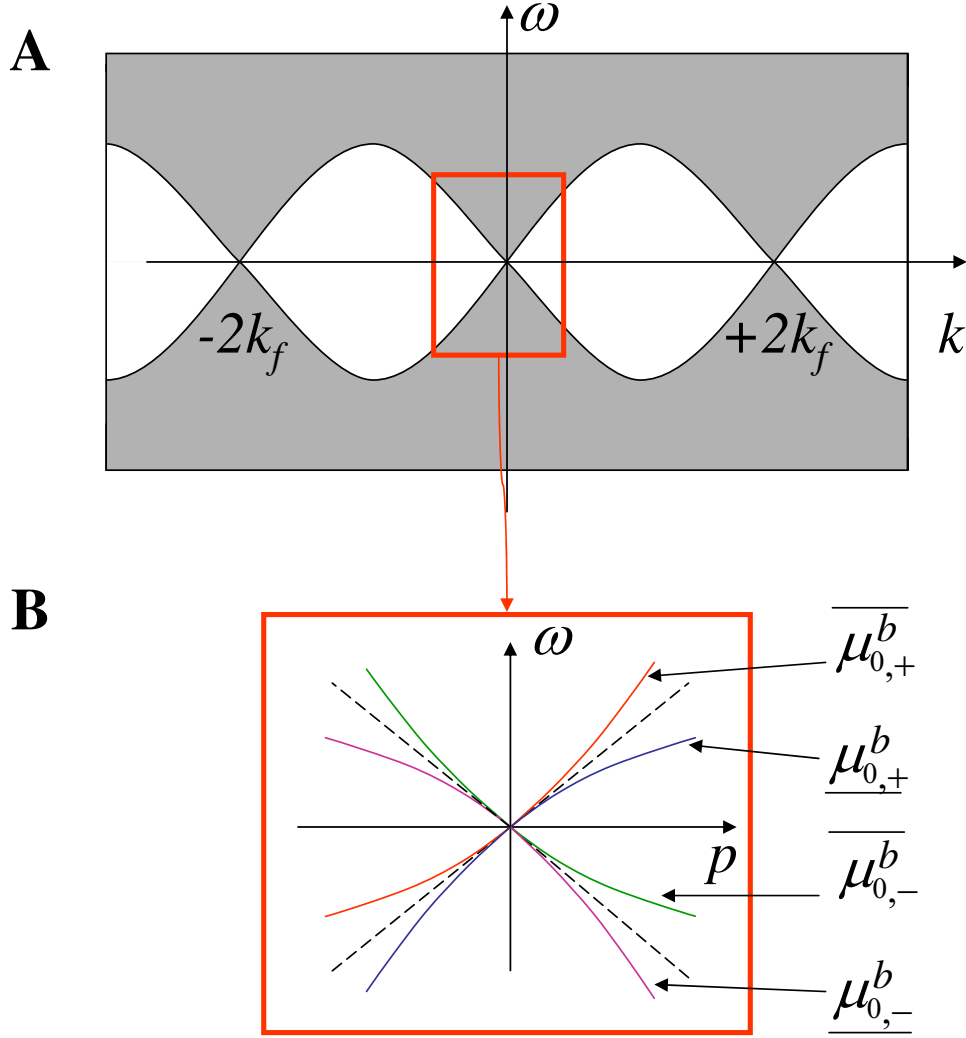


Figure S1: Bosonic spectral function. **(A)** Spectral function $A(k, \omega)$ in momentum-energy plane. Shaded areas indicate the regions where $A(k, \omega) \neq 0$. The region with $\omega > 0$ ($\omega < 0$) corresponds to the particle (hole) part of the spectrum. **(B)** Close-up view of the vicinity of $k \approx 0$, where $p = k$. Notations of μ indicate which exponents presented in Table S1 should be used in Eq. 6. Notations for exponents near $k \approx 2nk_f$ are obtained by substituting corresponding n instead of $n = 0$.

$\overline{\mu_{n,+}^b}$	$1 - \frac{1}{2} \left(2n - 1 - 2n \frac{\delta_+ + \delta_-}{2\pi} \right)^2 - \frac{1}{2} \left(\frac{\delta_+ - \delta_-}{2\pi} \right)^2$
$\mu_{n,+}^b$	$1 - \frac{1}{2} \left(2n + 1 - 2n \frac{\delta_+ + \delta_-}{2\pi} \right)^2 - \frac{1}{2} \left(2 - \frac{\delta_+ - \delta_-}{2\pi} \right)^2$
$\overline{\mu_{n,-}^b}$	$1 - \frac{1}{2} \left(2n + 1 - 2n \frac{\delta_+ + \delta_-}{2\pi} \right)^2 - \frac{1}{2} \left(\frac{\delta_+ - \delta_-}{2\pi} \right)^2$
$\mu_{n,-}^b$	$1 - \frac{1}{2} \left(2n - 1 - 2n \frac{\delta_+ + \delta_-}{2\pi} \right)^2 - \frac{1}{2} \left(2 - \frac{\delta_+ - \delta_-}{2\pi} \right)^2$

Table S1: Universal exponents for bosonic spectral function. Notations are indicated in Fig. S1, and parameters δ_{\pm} defined by Eq. 4 are functions of K only. Note that $\mu_{n,+}^b = \mu_{-n,-}^b$ which follows from the $k \rightarrow -k$ symmetry.

References

- S1. A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1963).
- S2. T. Giamarchi, *Quantum Physics in One Dimension* (Oxford Univ. Press, New York, 2004).
- S3. R. Blankenbecler, D.J. Scalapino, R.L. Sugar, *Phys. Rev. D* **24**, 2278 (1981).
- S4. J.E. Hirsch, *Phys. Rev. B* **31**, 4403 (1985).
- S5. I. Klich, in *Quantum Noise in Mesoscopic Systems*, Yu. V. Nazarov, Ed. (Kluwer, Dordrecht, 2003).
- S6. D.A. Abanin, L.S. Levitov, *Phys. Rev. Lett.* **93**, 126802 (2004).
- S7. D.A. Abanin, L.S. Levitov, *Phys. Rev. Lett.* **94**, 186803 (2005).
- S8. E.H. Lieb, W. Liniger, *Phys. Rev.* **130**, 1605 (1963).
- S9. A. Imambekov, L. I. Glazman, *Phys. Rev. Lett.* **100**, 206805 (2008).
- S10. V.V. Cheianov, M. Pustilnik, *Phys. Rev. Lett.* **100**, 126403 (2008).
- S11. R.G. Pereira *et al.*, *J. Stat. Mech.* P08022 (2007).
- S12. R. Chitra and T. Giamarchi, *Phys. Rev. B* **55**, 5816 (1997).
- S13. A.V. Rozhkov, *Phys. Rev. B* **74**, 245123 (2006).
- S14. R.G. Pereira *et al.*, *Phys. Rev. Lett.* **96**, 257202 (2006).
- S15. M. Pustilnik, M. Khodas, A. Kamenev, L.I. Glazman, *Phys. Rev. Lett.* **96**, 196405 (2006).

- S16. M. Khodas, M. Pustilnik, A. Kamenev, L.I. Glazman, *Phys. Rev. B* **76**, 155402 (2007).
- S17. F.D.M. Haldane, *Phys. Rev. Lett.* **60**, 635 (1988).
- S18. B.S. Shastry, *Phys. Rev. Lett.* **60**, 639 (1988).
- S19. B. Sutherland, *Beautiful Models* (World Scientific, Singapore, 2004).
- S20. M. Pustilnik, *Phys. Rev. Lett.* **97**, 036404 (2006).
- S21. A.V. Rozhkov, *Phys. Rev. B* **77**, 125109 (2008).
- S22. L.P. Pitaevskii, S. Stringari, *Bose-Einstein Condensation* (Oxford Univ. Press, New York, 2003).