No

Date

1月10日 [illegible]

Bosonization [illegible]

①: [illegible] 2D (or 1+1D) [illegible]　　t^2　x^2 [illegible]

一、Maxwell Equation.　　$\mathcal{L}=\frac{1}{2}(\dot{\vec{A}}^2-(\nabla\times\vec{A})^2)=\frac{1}{2}(\vec{E}^2-\vec{B}^2)$

2D: QHE　　$\vec{B}\perp$ [illegible]　$\vec{E}\parallel$ [illegible]

$(0,0,B)$　　$(E_x,E_y,0)$

$\vec{F}=e(\vec{E}+\vec{v}\times\vec{B}) \Rightarrow E=vB$　　$\dot{\theta}\propto E$.　$\dot{\theta}\propto E\times B$

2D　$\mathcal{L}=E_x^2+E_y^2-B_z^2$

(1+1)D　　$\mathcal{L}=E_x^2-B_z^2$　　$\vec{E}=\dot{\vec{A}}$　$\vec{B}=\nabla\times\vec{A}$　$\vec{A}$ vector

[illegible]　$\mathcal{L}=\dot{A}^2-(\partial_x A)^2$

☆ 1D [illegible]　　$\mathcal{L}=\frac{1}{2}[(\partial_t\phi)^2-u^2(\partial_x\phi)^2]$

☆ XY Model / Superfluid

$\mathcal{L}=\frac{1}{2}[(\partial_t\theta)^2-(\nabla\theta)^2] \overset{1d}{=} \frac{1}{2}[(\partial_t\theta)^2-(\partial_x\theta)^2]$

[illegible] A [illegible] θ, [illegible] φ [illegible] (Condensed Matter [illegible]

[illegible] $d=4$)

二、[illegible]　$\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{J}=0$.

(1+1)D　$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}J=0 \Leftrightarrow$ [illegible] $\begin{cases}\rho=\partial_x\phi\\ J=-\partial_t\phi\end{cases}$

三、[illegible]　$z=x+iy$ or $x=\bar{x}+it$

conformal symmetry [illegible]　$f(z)$. [illegible]

四、[illegible] chiral [illegible] $\phi(x\pm vt)$. [illegible] Fermion)

五、[illegible] ⇒ [illegible] ⇒ F, B [illegible] F (anyon), B [illegible]

[illegible] $d\geq 3$ [illegible] only exists Fermion and Boson [illegible]

六、linear spectrum / linear excitation: so there must be a cutoff

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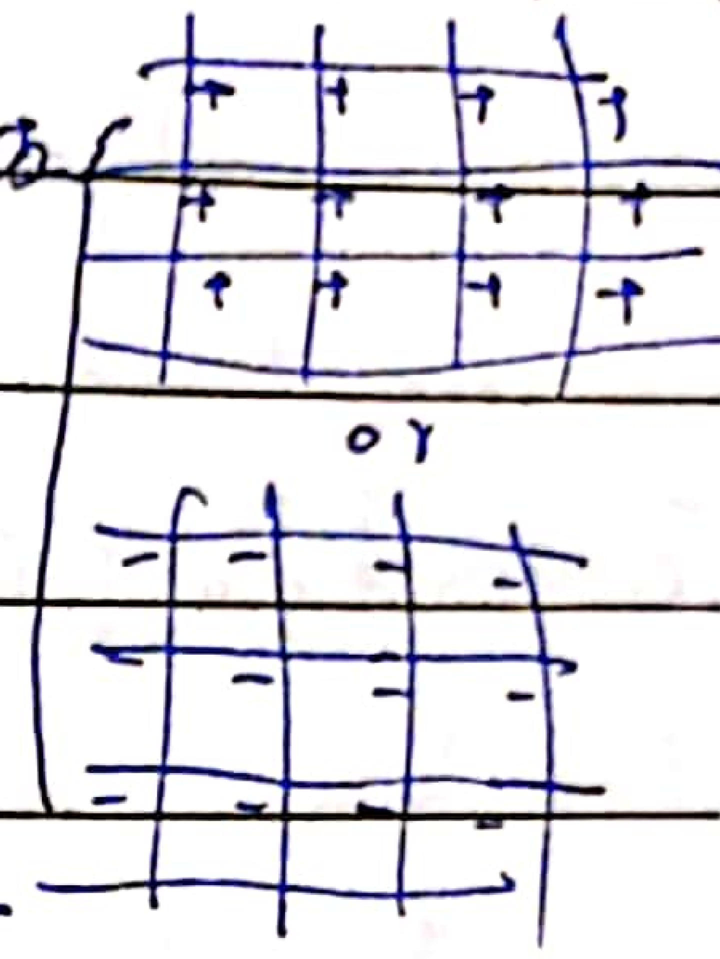
低温展开

$Z=\sum\prod e^{KS_iS_j}$

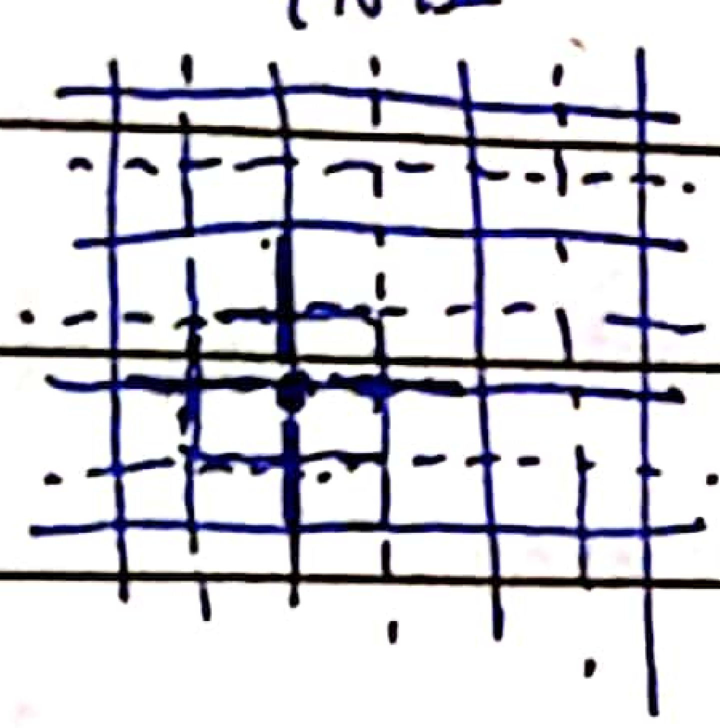
$J>0 \quad H=-J\sum_{ij}S_iS_j$ 基态



$2e^{K\varepsilon_0}+e^{K\varepsilon_0}N\,2e^{-4K}$ + 翻 + 记 四 …

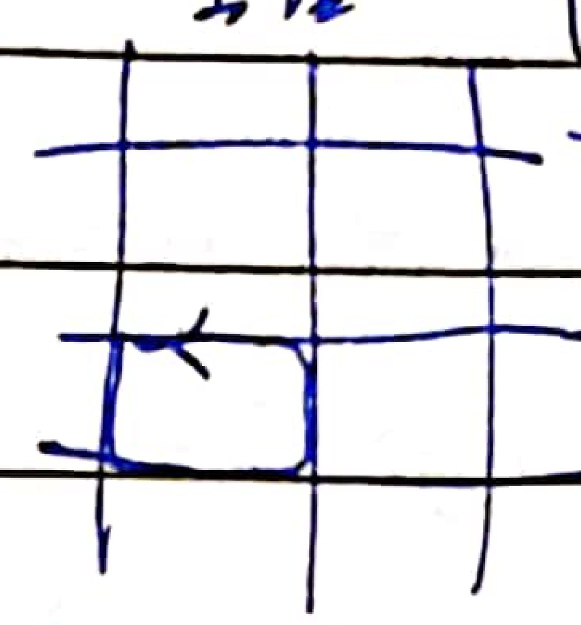
找出一个 Dual lattice

低温

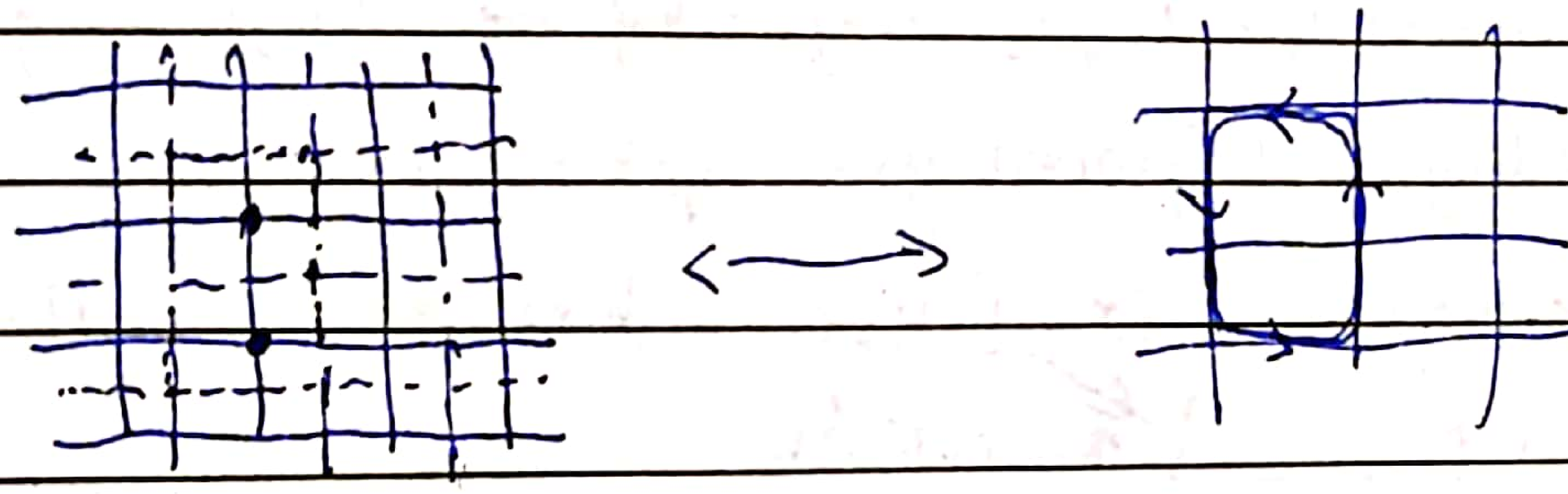


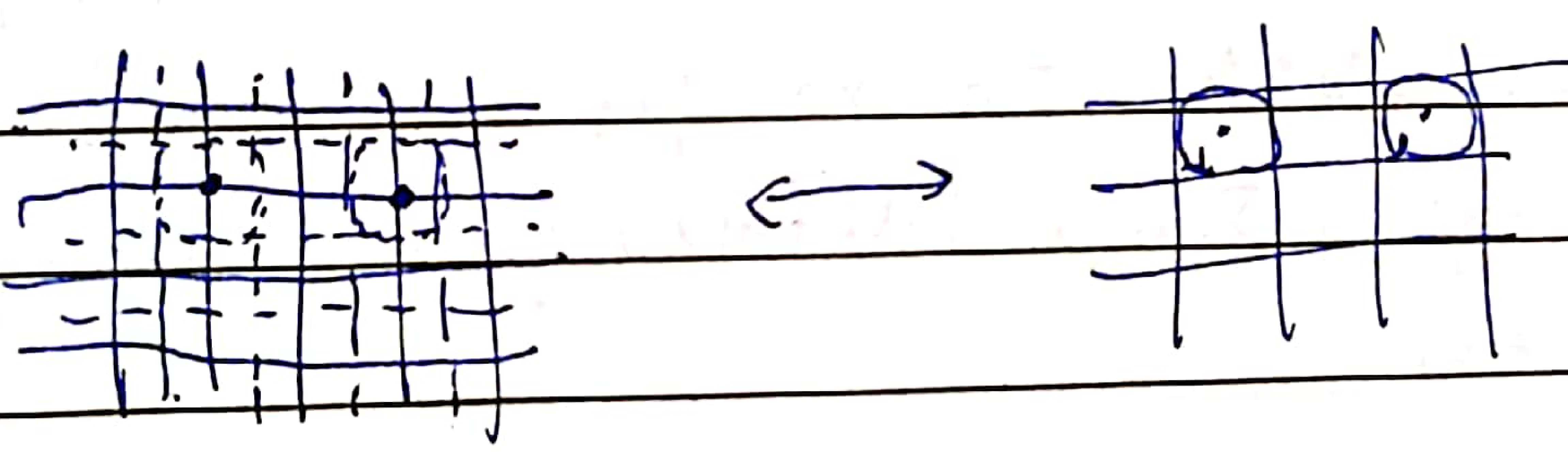
对应边 翻转 对应

高温



-- 对应





对于N $Z \propto f(e^{-2K})$ Slaughter 的 习题

$$\frac{Z_H}{2^N\cosh^{2N}K}=1+N\tanh^4(K)+2N\tanh^6(K)$$

$$\frac{Z_L}{e^{2NK}}=1+Ne^{-8K}+2Ne^{-12K}$$

故相变点在 $\tanh(K)=e^{-2K}$

$$K_c=\frac{1}{2}\ln(1+\sqrt{2})$$

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[illegible] (边) (右边定域) $\overset{a}{\circ\circ\circ\circ\circ\circ\circ\circ\circ}$ $x<a$ [illegible]

1. Jordan-Wigner 变换. Fermi-Boson [illegible] Anyon

2. Topo. term (Chern-Simons Term 等…)

[illegible]

F/A [illegible]

$b(x)$ [illegible] boson $\Rightarrow n_b = b^+ b$ $\quad f(x)$ fermion $n_f = f^+ f$.

$[b(x),\ b^+(y)b(y)] = b(x)b^+(y)b(y) - b^+(y)b(y)b(x)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \parallel$

$([b(x),\ b^+(y)] = \delta(x-y))$ $\quad (\delta(x-y) + b^+(y)b(x))b(y) - b^+(y)b(y)b(x)$

$= \delta(x-y)b(y) - b^+(y)b(x)b(y) - b^+(y)b(y)b(x)$

$= \delta(x-y)b(x)$ $\quad$ or $[n_b(y),\ b(x)] = -\delta(x-y)b(x)$.

[illegible]. $\hat{A}\psi = \lambda\psi$. $\quad$ [illegible]问题

[illegible] $[n_f(x),\ f(y)] = f^+(x)f(x)f(y) - f(y)f^+(x)f(x)$

$= f^+(x)f(x)f(y) - (\delta(x-y) - f^+(x)f(y))f(x) = -\delta(x-y)f(x)$

$= -\delta(x-y)f(x)$

[illegible]. $[n(x),\ \psi(y)] = -\delta(x-y)\psi(y)$ $\quad$ ψ [illegible] fermion or Boson [illegible]

Define. $b(x) = e^{i\phi(x)}$ $\quad$ ([illegible]) if it's true?

$[n(x),\ b(y)]$ [illegible] $b(y) = e^{i\phi(y)}$ $\quad$ $\phi(y)$ [illegible]

$\parallel$

$n(x)e^{i\phi(y)} - e^{i\phi(y)}n(x) = -\delta(x-y)e^{i\phi(y)}$

$e^{-i\phi(y)}n(x)e^{i\phi(y)} - n(x) = -\delta(x-y)$ $\quad \square$

[illegible]: Baker-Hausdorff-Feynman 公式

$e^{-A}Be^{A} = B - [A,B] + \frac{1}{2!}[A,[A,B]] - \cdots$

[illegible] $n(x) - i[\phi(y),\ n(x)] - n(x) = -\delta(x-y)$ $\quad$ $[n(x),\ \phi(y)] = i\delta(x-y)$

This is weird and not weird

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1.玻色化.

1.

Dirac Quantization.

$\psi = \sum_n c_n \varphi_n$ $\quad H = \langle\psi|H|\psi\rangle = t_{nm} c_n^* c_m$

$i\dot{c}_n = \frac{\partial H}{\partial c_n^*} \Rightarrow \dot{c}_n = \frac{\partial H}{\partial (i c_n^*)} \quad i\dot{c}_n^* = -\frac{\partial H}{\partial c_n}$

若 $c_n = \sqrt{r}e^{i\theta}$. 则 r 与 θ 也是共轭的 …… r 与 θ 对易.

$[b(x), b^+(y)] = \delta(x-y)$, $b = e^{i\phi}\sqrt{n}$. $b^+ = \sqrt{n}e^{i\phi}$.

$b^+b = n \quad bb^+ = e^{i\phi} n e^{-i\phi} \quad [b(x), b^+(y)] = e^{i\phi} n e^{-i\phi} - n$

$= \delta(x-y) \quad [\phi, n] = i\delta(x-y)$ (这里忽略了 …… $b^+b = \sqrt{n(x)} e^{i\phi(x)} e^{-i\phi(y)} \sqrt{n(y)}$

费米子, $b^+(x) b(x) = e^{i\phi(x)} e^{-i\phi(x)} = 1 \neq n(x)$

…… $\frac{\partial\rho}{\partial t} + \frac{\partial J}{\partial x} = 0 \quad \rho \propto \partial_x \phi$

問題: Fermion $\Rightarrow$ 用 n, ϕ 描述. n 与 ϕ 是 Boson

1934. 1d fermions have the same type of low-energy excitations as harmonica ~~(Boson)~~ chain (Bosons) $\Rightarrow$ sound wave

1950. Tomonaga: remarks of Bloch's method of sound waves applied to many-fermion problem (Mahan 书)

1964. Luttinger $\rightarrow$ 严格可解的 model. …… 运动 ~~X~~

4. Duncan Haldane 1981 (……)

ref: celebrating Haldane's luttinger liquid theory

ref: Shankar 书. 17-18 章 B. Simons 书 第二章 (6页)

XG Wen, edge state in hydrodynamic description

ρ 描述

lectures on Bosonization by C. L. Kane

問題: …… $e^{i\phi}$ ϕ: Boson field 其 …: Abel Jordan-Wigner

…… JW 变换 …… Bloch's Sound wave

$\psi \Rightarrow \frac{\eta}{\sqrt{2\pi a}} e^{i\phi}$ $\quad \begin{cases} \eta: \text{Klein operator} \\ a: \text{cut off} \end{cases}$

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Jordan-Wigner 变换. (Shankar 第八章)

Lattice Model. ~~$f_i = e^{i\theta_i} b_i$~~

$f_i = e^{i\theta_i} b_i$ $\quad f_i f_j = e^{i\theta_i} b_i e^{i\theta_j} b_j \quad f_j f_i = e^{i\theta_j} b_j e^{i\theta_i} b_i$

~~$e^{i\theta_i} e^{i\theta_j} b_i e^{-i\theta_j}$~~ $\quad e^{i\theta_i} e^{i\theta_j} e^{-i\theta_j} b_i e^{i\theta_j} b_j \quad e^{i\theta_j} e^{i\theta_i} e^{-i\theta_i} b_j e^{i\theta_i} b_i$

对于 $i<j$ $\quad$ i — j $\quad$ 且 θ_i 和 i 有关 b 有关. 即 $[\theta_i, b_j]=0$

$f_j f_i = e^{i\theta_j} e^{i\theta_i} b_i$ $\quad$ 让 $f_i f_j$ 反对易

$e^{-i\theta b^+ b} b e^{i\theta b^+ b} |n\rangle = \sqrt{n} e^{in\theta} e^{-i\theta b^+ b} |n-1\rangle = e^{i\theta} b |n\rangle$

$f_i f_j = e^{i\theta_i} e^{i\theta_j} (e^{i\theta} + 1) = 0$ $\quad$ String

$f(x) = e^{i\pi \int_{-\infty}^{x} b(y) b(y) dy} b(y)$

Anyon: $\quad f(x) f(y) = e^{i\theta} f(y) f(x) \quad \theta$ 任意取 → Anyon

$f_i = e^{i\theta_i} b_i \quad f_i^2 = 0 \Leftrightarrow e^{i\theta_i} b_i e^{i\theta_i} b_i \propto 0 \quad b_i^2 \neq 0$

If $f = e^{i\theta} b \Rightarrow H_F \to H_B \quad Z_F = Z_B$

玻色. Duality. (Haldane)

中心荷 (Mark) $\quad C_L = \frac{\pi}{3} k_B \frac{k_B T}{c_s} \quad$ 费米 $\quad C_L = \frac{\pi}{3} k_B \frac{k_B T}{\hbar v_F}$

FQHE → Anyon in Magnetic $\quad \psi(x) = e^{i \int \vec{A} \cdot d\vec{l}} \psi(x)$

General ① $\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial x} = 0 \quad \rho = \partial_x \phi \quad j = -\partial_t \phi \quad$ ② $[n, \phi] = i\delta(x-x')$

即 Density 与 phase 共轭 $\quad$ ③ $f(x) = e^{i\pi \int_{-\infty}^{x} \rho(x') dx'} b(x)$

$f(x) = e^{i\pi \int_{-\infty}^{x} \rho(x') dx'} b(x) = e^{i\pi \int_{-\infty}^{x} \rho(x') dx'} \sqrt{n} e^{i\theta(x)}$

$e^{i(\pi \int_{-\infty}^{x} \rho(x') dx' + \theta(x))}$ $\quad$ 远处的不变

Thus: $\phi = \pi \int_{-\infty}^{x} \rho(x') dx' + \theta(x)$ $\quad$ 有涨落: 密度的起伏

$\rho = \rho_0 + \delta\rho$ $\quad$ 则 $\phi = \pi \rho_0 x + \pi \int_{-\infty}^{x} \delta\rho(y) dy + \theta(x)$

$\rho_0 = \frac{N}{L}, \; n = \frac{1}{L} \sum_k = \frac{1}{2\pi} \int_{-k_F}^{k_F} dk = \left(\frac{k_F}{\pi}\right)$ $\quad \rho_0 = ? \quad$ 一定记住 $k_F x$

$+ \pi \int_{-\infty}^{x} \delta\rho(y) dy + \theta(x)$

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[illegible] ② [illegible] $\pi \int_{-\infty}^{x} \delta\rho(y)\, dy$ ③ [illegible]

[illegible] Local [illegible] ↓ [illegible] string

Maxwell Equation

$$\begin{cases} \frac{\partial E}{\partial x} = -\frac{\partial H}{\partial t} & E = \dot{A} \quad H = -\partial_x A \\ \frac{\partial E}{\partial t} = -\frac{\partial H}{\partial x} & E = \partial_x \phi \quad H = -\left(\frac{\partial \phi}{\partial t}\right) \end{cases}$$

[illegible] Shankar [illegible], chap 17.

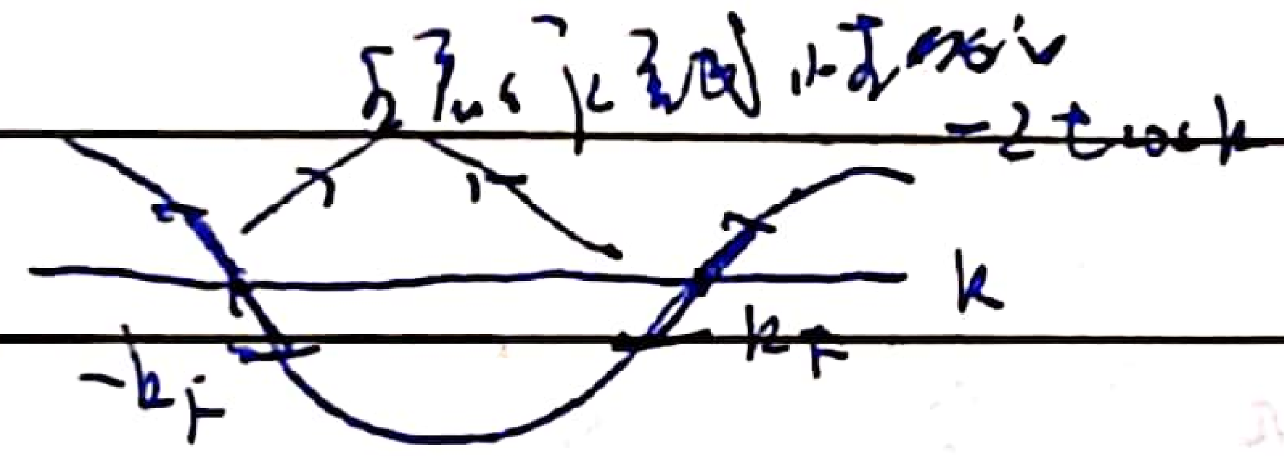
Bosonization: Fermions $\to$ Boson field.

$\psi \sim e^{i\phi}$

([illegible] $\int D\bar{\psi} D\psi\, e^{-S[\bar{\psi},\psi,A]} = \int DA\, e^{-S_{eff}[A]}$ [illegible])

Lattice Model

$$H = -\sum_n (\psi_n^+ \psi_{n+1} + h.c.) + h \psi_n^+ \psi_n = \sum_k (-2t\cos k + h) \psi_k^+ \psi_k$$



[illegible] $\int dx\, \psi^+ \left(-\frac{\partial^2}{\partial x^2} + h\right) \psi$ the FS [illegible] Dirac Equation

[illegible] ψ_L [illegible] ψ_R. $H = \sum_k v_F (k - k_F) \psi_R^+(k) \psi_R(k) + \sum_k v_F (k + k_F) \psi_L^+(k) \psi_L(k)$

$\psi_R(k) \to \psi_{R}(k+q) \triangleq \psi_R(q)$ ψ_L [illegible].

$$H = \sum_{q>0} v q\, \psi_R^+(q) \psi_R(q) - \sum_{q>0} v q\, \psi_L^+(q) \psi_L(q)$$

$$\psi_q = \begin{pmatrix} \psi_R(q) \\ \psi_L(q) \end{pmatrix} \qquad H_{eff} = \psi_q^+ \begin{pmatrix} vq & 0 \\ 0 & -vq \end{pmatrix} \psi_q$$

$\psi = \frac{1}{\sqrt{2\pi a}} e^{\pm i\sqrt{4\pi}\phi(x)}$ eq 17.72 (Shankar)

Remarks. Not an operator Identity. [illegible] 和 Boson 的 [illegible]
([illegible] Boson [illegible], 100% [illegible] Fermi [illegible])

Fermi [illegible] Bose [illegible] a cutoff

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Bosonization

$$\psi_\pm(x) = \frac{1}{L}\sum_k \psi_\pm(k) e^{ikx} e^{-\frac{1}{2}\alpha|k|}$$

$$= \frac{1}{\sqrt{\pi}}\int dk\, \psi_\pm(k) e^{ikx} e^{-\frac{1}{2}\alpha|k|}$$

$\{\psi_\pm(x), \psi_\pm(y)\} = 0$

$$\langle \psi_+(x)\psi_+^\dagger(0)\rangle = \left(\frac{1}{\sqrt{\pi}}\right)^2 \int dp\,dq\, e^{ipx+iq0} e^{-\frac{1}{2}\alpha|p| - \frac{1}{2}\alpha|q|} \quad \psi_+(p)\psi_+^\dagger(q) \to 2\pi\delta(p-q)$$

if $|k|<k_F$. $a_k^+|\cdots\rangle = 0$ $\quad C_k|\cdots\rangle \neq 0$ $\quad q>0 \quad p=q$

$$\langle \psi_+(x)\psi_+^\dagger(0)\rangle = \frac{1}{(\sqrt{\pi})^2}\int_0^{+\infty} e^{ipx-\alpha|p|}dp = \frac{1}{\sqrt{\pi}}\frac{1}{\alpha - ix}$$

① $\langle \psi_+(x)\psi_+^\dagger(0)\rangle = \frac{1}{\sqrt{\pi}}\frac{1}{\alpha - ix}$

② $\langle \psi_+^\dagger(0)\psi_+(x)\rangle = \left(\frac{1}{\sqrt{\pi}}\right)^2 \int dp\,dq\, e^{ipx+iq0-\frac{1}{2}\alpha|p|-\frac{1}{2}\alpha|q|} \langle \psi_+^\dagger(q)\psi_+(p)\rangle$

$$= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{0} dp\, e^{ipx+\alpha p}$$

$$①+② = \frac{1}{\sqrt{\pi}}\frac{1}{\alpha-ix} + \frac{1}{\sqrt{\pi}}\frac{1}{\alpha+ix} = \frac{\alpha/\pi}{\alpha^2+x^2} = \delta(x) \quad \text{if } \alpha\to 0 \quad (x\gg\alpha)$$

$\langle \psi(x)\psi^\dagger(0)\rangle = \frac{1}{\sqrt{\pi}}\frac{1}{\alpha-ix}$ $\quad$ 若 $\psi = e^{i\phi(x)}$.

$\langle e^{i\phi(x)} e^{-i\phi(0)}\rangle \sim \frac{1}{\sqrt{\pi}}\left(\frac{1}{\alpha-ix}\right)$ 找一个 Boson field ϕ

Jordan-Wigner $\pi\int_{-\infty}^{x} b^\dagger(x)b(x)\,dx$

Scalar (Boson) Field:

$$H_0 = \frac{1}{2}\int \Pi^2 + (\partial_x\phi)^2\, dx. \quad \mathcal{L} = \frac{1}{2}\int (\partial_t\phi)^2 - (\partial_x\phi)^2$$

$$\Pi = \frac{\partial\mathcal{L}}{\partial\dot\phi} = \partial_t\phi$$

正则量子化. $[\phi, \partial_t\phi] = i\delta(x-x')$

$$\phi = \int \frac{dp}{\sqrt{\pi}\sqrt{2|p|}}\left(\phi(p)e^{ipx} + \phi^\dagger(p)e^{-ipx}\right)e^{-\frac{1}{2}\alpha|p|}$$

$\phi(x+t) \quad \phi(x-t) \qquad px = \vec{p}\cdot\vec{x} - |p|t$

$$\Pi = \partial_t\phi = \int \frac{dp\,|p|}{\sqrt{\pi}\sqrt{2|p|}}\left(-i\phi(p)e^{ipx} + i\phi^\dagger(p)e^{-ipx}\right)e^{-\frac{1}{2}\alpha|p|}$$

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Pline. $[\phi(p), \phi^\dagger(p')] = 2\pi\delta(p-p')$

$$[\phi(x), \Pi(y)] = \frac{i\alpha/\pi}{\alpha^2+(x-y)^2} = i\delta(x-y)$$

$$\int \frac{dp}{2\pi} |p| \phi^\dagger(p)\phi(p)$$

引入 right mover left mover.

$$\phi_+(x) = \frac{1}{2}\left[\phi(x) - \int_{-\infty}^{x} \Pi(x')dx'\right]$$

$$\phi_-(x) = \frac{1}{2}\left[\phi(x) + \int_{-\infty}^{x} \Pi(x')dx'\right]$$

$$\begin{cases} \phi_+ + \phi_- = \phi \\ \phi_+ - \phi_- = -\int_{-\infty}^{x} \Pi(x')dx' \end{cases}$$

$$\phi_+ = \frac{1}{2}\int_0^{+\infty} \frac{dp}{2\pi\sqrt{2|p|}} \left(e^{ipx}\phi(p) + e^{-ipx}\phi^\dagger(p)\right) e^{-\frac{1}{2}\alpha|p|}.$$

$$\phi_- = \frac{1}{2}\int_{-\infty}^{0} \frac{dp}{2\pi\sqrt{2|p|}} \left(e^{ipx}\phi(p) + e^{-ipx}\phi^\dagger(p)\right) e^{-\frac{1}{2}\alpha|p|}.$$

$\phi = \phi_+ + \phi_-$ $\qquad \psi(x) = e^{i\phi(x)}$ $\qquad \underline{e^{i\phi(x)}e^{i\phi(y)} + e^{i\phi(y)}e^{i\phi(x)} = 0}$

$$\begin{cases} \psi(x)\psi(y) + \psi(y)\psi(x) = 0 \quad x \neq y \text{ or } |x-y| \gg \alpha. \\ \langle \psi_+(x)\psi_+^\dagger(0)\rangle = \frac{1}{2\pi}\frac{1}{\alpha + i x} \end{cases}$$

$$e^{-A}Be^{A} = B - [A,B] - \cdots$$

$$e^{A}e^{B} = e^{A+B+\frac{1}{2}[A,B]}$$

$$= e^{i\phi(x)+i\phi(y)-\frac{1}{2}[\phi(x),\phi(y)]} + e^{i\phi(x)+i\phi(y)+\frac{1}{2}[\phi(x),\phi(y)]} = 0$$

$$[\phi(x), \phi(y)] = \pi\,\mathrm{sgn}(x-y) \qquad [\phi_+, \phi_-] = \frac{i}{4}$$

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