Berry phase effect on magnon Hall effect

Anomalous Hall effect is well-know in electronic system. Moreover, there is also Anomalous Hall effect in magnon system. Unlike electrons, magnon does not carry charge so it will not feel electric field. But when we apply a temperature gradient, the statistical force will drive the magnon and cause the magnon Hall effect.

In an anti-ferromagnetic honeycomb lattice in $x-y$ plane with an external magnetic field in z direction, the nearest DM interaction is 0 and the next nearest DM interaction is in the z direction[3]. Calculate the Berry curvature and magnon Hall current.

Firstly, you should write down the spin Hamiltonian of this system

$$H = J\sum_{\langle ij\rangle}\boldsymbol{S_i}\cdot\boldsymbol{S_j} + \sum_{\langle\langle ij\rangle\rangle}\boldsymbol{D}_{ij}\cdot(\boldsymbol{S_i}\times\boldsymbol{S_j}) + \mathcal{K}\sum_i S_{iz}^2 + \sum_i -\boldsymbol{\mu}\cdot\boldsymbol{B} \tag{1}$$

Make sure you know the meaning of every term in the Hamiltonian above. By means of the Holstein-Primakoff transformation, you can turn the Hamiltonian into second quantization form

$$S_i^+ = \sqrt{2S}a_i, \quad S_i^- = \sqrt{2S}a_i^\dagger, \quad S_i^z = S - a_i^\dagger a_i \tag{2}$$

then do the Fourier transformation and use Nambu basis to express the spin Hamiltonian

$$H = \psi_k^\dagger \mathcal{H}_k \psi_k \tag{3}$$

$$\psi_k = \begin{pmatrix} a_k \\ b_k^\dagger \end{pmatrix}, \quad \begin{pmatrix} a_i \\ b_i^\dagger \end{pmatrix} = \frac{1}{\sqrt{N}}\sum_k e^{ik\cdot R_i}\begin{pmatrix} a_k \\ b_k^\dagger \end{pmatrix} \tag{4}$$

Secondly, you should use Bogoliubov transformation

$$\begin{pmatrix} \alpha_k \\ \beta_k^\dagger \end{pmatrix} = \begin{pmatrix} u_k & -v_k \\ -v_k & u_k \end{pmatrix}\begin{pmatrix} a_k \\ b_k^\dagger \end{pmatrix} \tag{5}$$

to rewrite the Hamiltonian and get the equation of motion of the new quasiparticle operators

$$i\hbar\dot{\alpha}_k = [\alpha_k, \mathcal{H}_k] \tag{6}$$

After obtaining the eigenequation, you can just solve it to get the energy spectrum and eigenvectors. (Why not just diagonalize the matrix form of H in Nambu basis?About this you can read the reference[4])

Next, you should use the energy spectrum and eigenvectors you have got to calculate the Berry curvature of magnon. Take care of the σ_z when you deal with the magnon dynamics. So the Lagrangian will be like

$$\mathcal{L} = \left\langle W \middle| i\hbar\sigma_z(d/dt) - \mathcal{H}' \middle| W \right\rangle \tag{7}$$

$$\mathcal{H}' = \mathcal{H}_k + U(r) \tag{8}$$

where $U(r)$ is the potential felt by magnon.

Finally, consider the potential-driven magnon Hall effect and get the thermal conductivity. The dynamics of the wave packet is described by the semiclassical equation of motion[1] [2]

$$\begin{aligned} \dot{\boldsymbol{r}} &= \frac{1}{\hbar}\frac{\partial \varepsilon_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}_n(\boldsymbol{k}), \\ \hbar\dot{\boldsymbol{k}} &= -\nabla U(\boldsymbol{r}) \end{aligned} \tag{9}$$

You may derive the thermal conductivity through

$$\begin{aligned} j_Q &= \int d\varepsilon f(\varepsilon)\varepsilon\dot{r} \\ j_{Qx} &= \sigma_{xy}\partial_y U(r) \end{aligned} \tag{10}$$

where $f(\varepsilon)$ is Bose-Einstein distribution, ε is energy of magnon, and σ_{xy} is thermal conductivity driven by potential.

Reference:

[1] Ryo Matsumoto and Shuichi Murakami, Rotational motion of magnons and the thermal Hall effect, Phys. Rev. B 84, 184406 (2011).

[2] Ryo Matsumoto and Shuichi Murakami, Theoretical Prediction of a Rotating Magnon Wave Packet in Ferromagnets, Phys. Rev. Lett.106, 197202 (2011).

[3] Ran Cheng, Satoshi Okamoto, and Di Xiao, Spin Nernst Effect of Magnons in Collinear Antiferromagnets, Phys. Rev. Lett.117, 217202 (2016).

[4] J. H. P. Colpa, Physica 93A, 327 (1978).