

4.30 $k_B T_c = \frac{t}{2} J$ here, pairing is fun.

$$\sum_{n=-\infty}^{+\infty} f(n) = \sum_{k=-\infty}^{+\infty} f(k)$$

if null : $\tilde{f}(k) = \sum_n f(n) e^{2\pi i n k}$ $\sum_k \tilde{f}(k) = \sum_n f(n) \sum_k e^{2\pi i n k}$ non-zero

$$\sum_n f(n) \sum_m \delta(n-m) = \sum_m f(m)$$

eg. $f(n) = e^{-an^2 - bn}$ $\tilde{f}(k) = \sqrt{\frac{\pi}{a}} e^{(b - 2\pi i k)^2 / 4a} = \sqrt{\frac{\pi}{a}} e^{-(2\pi k - ib)^2 / 4a}$

e.g. $a=1$, $b=0.53$.

Ref $\sum_n f(n) = \sum_k \tilde{f}(k) = 1.0138$

4.4.1 $f(n)$ 中 n 的取值 $\tilde{f}(k)$ 中 k 的取值 两者都是 2π 的周期函数

$f(n)$ 的周期 $\tilde{f}(k)$ 的周期 $\Rightarrow$ 它们互为傅里叶变换。

$\alpha \times \beta$ 的取值 $\tilde{f}(k)$ 的取值

$\tilde{f}(k)$ 的取值 $\tilde{f}(k)$ 的取值 $\tilde{f}(k)$ 的取值

Solar Simulation

$$\sum_{n=1}^k f(n) \rightarrow \int_1^k f(x) dx + \dots$$

BKT 的推导

0. $F = U - TS = J \ln \frac{1}{2} - k_B T \ln \frac{1}{2} = (J - k_B T) \ln \frac{1}{2}$

$k_B T_c = \frac{J}{2} \Rightarrow$ 2-3 的取值

1. Thouless Nelson-Kosterlitz (PRL)

2. Wilson. RG. ref. Xiao Wan. Shankar Sen

3. Jinn (S. Jinn & dno) Nagasawa.

BKT. RG: (why 3m, 4m)

$$\langle e^u \rangle = e^{\langle u \rangle + \frac{1}{2} \langle u^2 \rangle} \quad \text{if } \frac{1}{2} \langle e^{J u} \rangle = e^{\langle J u \rangle + \frac{1}{2} \langle J^2 u^2 \rangle}$$

$$\langle u \rangle = \langle e^k \rangle = e^{\langle k \rangle + \frac{1}{2} \langle k^2 \rangle}$$

$$u = \omega \theta = e^{i\theta} + e^{-i\theta}$$

$$u = \sqrt{2} e^{i\theta}$$

$$\langle u \rangle = e^{\langle i A \omega \theta \rangle} = e^{\frac{i A}{2} (\langle e^{i\theta} \rangle + \langle e^{-i\theta} \rangle)} = e^{\frac{i A}{2} (e^{i\theta - \frac{1}{2} \langle \theta^2 \rangle} + e^{-i\theta - \frac{1}{2} \langle \theta^2 \rangle})}$$

$$= e^{i A} e^{-\frac{1}{2} \langle \theta^2 \rangle}$$

That's Fucking Amazing

No

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$$\cos n(\theta + \theta_c) - \cos n\theta_c$$

$$= \cos n\theta_c \cos n\theta_c - \sin n\theta_c \sin n\theta_c - \cos n\theta_c$$

$$= \cos n\theta_c \left(1 - \frac{1}{2} n^2 \theta_c^2\right) - n\theta_c \sin n\theta_c$$

$$S = \int \frac{k}{2} (\partial_\mu \theta_c)^2 - g \cos n\theta_c + \int \frac{k}{2} (\partial_\mu \theta_c)^2 - g (\cos n(\theta_c + \theta) - \cos n\theta_c)$$

$$= \int \frac{k}{2} (\partial_\mu \theta_c)^2 - g \cos n\theta_c + \int \frac{k}{2} (\partial_\mu \theta_c)^2 + g \frac{n^2}{2} \cos n\theta_c \theta_c^2 + n g \sin n\theta_c \theta_c$$

$$Z = \int D\theta_c e^{-\left[\frac{k}{2} (\partial_\mu \theta_c)^2 - g \cos n\theta_c\right]} \int D\theta_c e^{-\left[\frac{k}{2} (\partial_\mu \theta_c)^2 - g \left(\frac{n^2}{2} \cos n\theta_c \theta_c^2 + n \sin n\theta_c \theta_c\right)\right]}$$

$$\theta \leq e^u = e^{cu + \frac{1}{2} cu^2}$$

$$- \langle \theta^2 \rangle$$

$$e^{-g \frac{n^2}{2} \langle \theta_c^2 \rangle} \cos n\theta_c$$

$$\langle \theta_c^2 \rangle$$

$$= g^2 n^2$$

$$\sin n\theta_c(x) \sin n\theta_c(x+y) g(x+y)$$

$$= g^2 n^2 \int \sin n\theta_c(x) \sin n\theta_c(x+y) g(x+y)$$

$$= g^2 n^2 \int \sin n\theta_c(x) [\sin n\theta_c(x) + \partial_\mu \theta_c(x)] g(x+y)$$

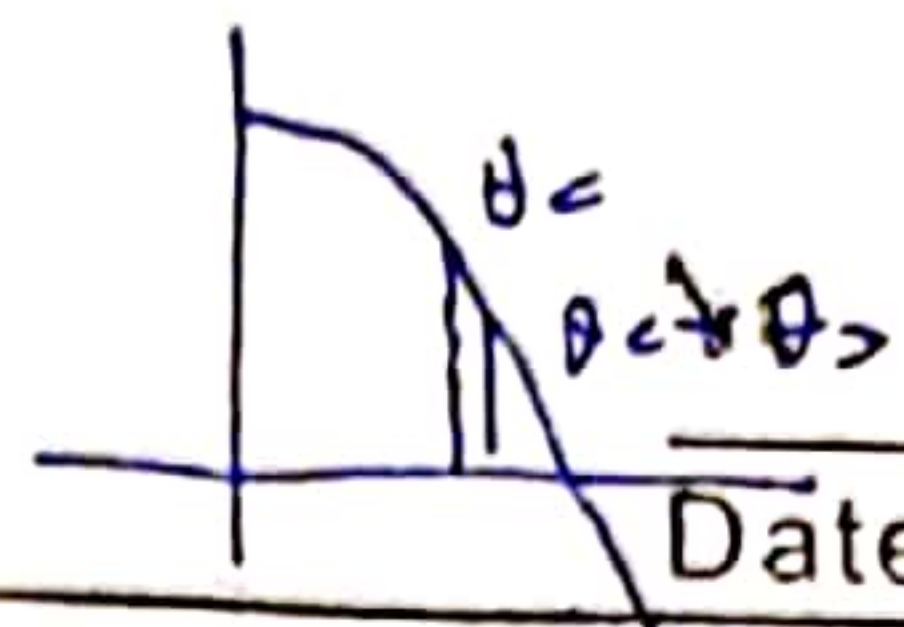
$$= g^2 n^2 \int \sin n\theta_c(x) \left\{ \sin n\theta_c(x) \cos n\theta_c y + \cos n\theta_c(x) \sin n\theta_c y \right\} g(y)$$

$$= g^2 n^2 \int \sin n\theta_c(x) \left\{ \sin n\theta_c(x) \left[1 - \frac{1}{2} n^2 \theta_c^2 y^2 \right] + \cos n\theta_c(x) \sin n\theta_c y \right\} g(y)$$

$$= \dots - \frac{1}{2} g^2 n^4 \int \sin^2 n\theta_c(x) (\partial_\mu \theta_c)^2 y^2 g(y)$$

$$= \dots - \frac{1}{4} g^2 n^4 \int (1 - \cos 2n\theta_c(x)) (\partial_\mu \theta_c)^2 y^2 g(y)$$

$$= \dots - \frac{1}{4} g^2 n^4 \int (\partial_\mu \theta_c)^2 y^2 g(y)$$



x, y, z, w, u, v, p, q, r, s, t, u, v, w, x, y, z, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 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980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000

7/11/18

$$\theta = \theta_c + \theta_0$$

$$S = \int \frac{K}{2} (\partial_\mu \theta_c)^2 - g \cos \theta_c + \int \frac{K}{2} (\partial_\mu \theta_0)^2 - g [\cos(\theta_c + \theta_0) - \cos \theta_c]$$

$$= \int \frac{K}{2} (\partial_\mu \theta_0)^2 + g \frac{n^2}{2} \cos n \theta_c \theta_0^2 - n g \sin \theta_c \theta_0$$

perturbation

$$Z = \int D\theta_c e^{-S_c} \int D\theta_0 e^{-S_0} \left\langle e^{-\int \frac{K}{2} (\partial_\mu \theta_0)^2 - g \int \frac{n^2}{2} \cos n \theta_c \theta_0^2 - n g \sin \theta_c \theta_0} \right\rangle$$

$$- \frac{1}{Z} \frac{\delta Z}{\delta \theta_c} = \langle \partial_\mu \theta_0 \rangle = 0 \quad \langle \partial_\mu \theta_0 \theta_0 \rangle = g \cos \theta_c$$

$$\begin{aligned} &= \frac{1}{Z} \frac{\delta Z}{\delta \theta_c} \\ &= \frac{1}{Z} \frac{\delta}{\delta \theta_c} \int D\theta_c D\theta_0 e^{-S_c - S_0} \\ &= \frac{1}{Z} \int D\theta_c D\theta_0 \left(\frac{\delta S_c}{\delta \theta_c} + \frac{\delta S_0}{\delta \theta_c} \right) e^{-S_c - S_0} \\ &= \frac{1}{Z} \int D\theta_c D\theta_0 \left(-\frac{K}{2} \partial_\mu^2 \theta_c + g \sin \theta_c \right) e^{-S_c - S_0} \\ &= \frac{1}{Z} \int D\theta_c D\theta_0 \left(-\frac{K}{2} \partial_\mu^2 \theta_c + g \sin \theta_c \right) e^{-S_c - S_0} \end{aligned}$$

$$g^2 n^2 \int \cos(n \theta_c(x)) \cos(n \theta_c(x')) g_{xx-x'} \quad g^2 n^2 \int \cos(n \theta_c(x)) \cos(n \theta_c(x+y)) g_{xy}$$

for the y part we can use the identity

$$\cos n \theta_c(x) \cos n \theta_c(x+y) = \frac{1}{2} [\cos n \theta_c(x) + \cos n \theta_c(x+y)] \cos n \theta_c(x+y)$$

$$= \cos n \theta_c(x) \cos n \theta_c(x+y) \cdot \left[1 - \frac{n^2}{2} y^2 (\partial_\mu \theta_c)^2 \right] - \sin n \theta_c(x+y)$$

$$\int \cos n \theta_c(x) \cos n \theta_c(x+y) g_{xy} dy = \int \cos n \theta_c(x) \cos n \theta_c(x+y) g_{xy} dy$$

$$= \int \cos n \theta_c(x) \cos n \theta_c(x+y) g_{xy} dy = \int \cos n \theta_c(x) \cos n \theta_c(x+y) g_{xy} dy$$

$$= \int \cos n \theta_c(x) \cos n \theta_c(x+y) g_{xy} dy = \int \cos n \theta_c(x) \cos n \theta_c(x+y) g_{xy} dy$$

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$$z \rightarrow z_1: Z = \int p \theta_c e^{-\int \left(\frac{k^2}{2} + \frac{g^2}{8} n^2 \right) y^2 g(y) dy} (\partial \theta_c) = g(1 - \frac{n^2}{2} \langle \theta^2 \rangle) \cos n \theta_c$$

$$S = \int dx \frac{k^2}{2} (\partial \theta_c)^2 - g \cos n \theta_c$$

$$k_2 = k + \frac{g^2}{8} n^2 \int y^2 g(y) dy \Rightarrow \begin{cases} g_2 = g - \frac{1}{2} g \langle \theta^2 \rangle \\ k_2 = k + \frac{n^2}{8} g^2 k_2 \end{cases}$$

$$g_2 = g - \frac{n^2}{2} g \langle \theta^2 \rangle$$

$$k(\omega) = \int_{2\pi/\lambda < |k| < 2\pi/\lambda} \frac{dk}{2\pi} \frac{n^2}{|k| |k'|} = \frac{n^2}{2\pi k} \int \frac{dk}{k} = \frac{n^2}{2\pi k} \ln \frac{\lambda}{l}$$

$$R_2 = \frac{1}{d^2} \frac{d^2}{dx^2} |x|^2 k(x) = \int_{2\pi/\lambda < |k| < 2\pi/\lambda} \frac{d^2}{dx^2} \frac{dk^2}{k^2} \frac{n^2 x^2 e^{i k x + \omega t}}{k_1 k_2}$$

$$= \frac{\lambda - l}{l} \frac{3n^2 l^4}{16\pi k} \int d\theta \frac{1}{(\cos \theta + i \sin \theta)^4} = \frac{\lambda - l}{l} \frac{3n^2 l^4}{2\pi k}$$

$$x' = bx$$

$$b = \frac{l}{\lambda} \quad 0 < |x| < \lambda \quad 1 < |x'| < l$$

$$\int dx \frac{k^2}{2} (\partial \theta_c)^2 - g \cos n \theta_c = \frac{k_2}{2} \int dx' \frac{1}{b^2} b^2 (\partial \theta_c)^2$$

$$- g \frac{1}{b^2} \int dx' \cos n \theta_c(x')$$

$$\frac{k}{2} \langle \theta^2 \rangle = \frac{1}{2} (k + \frac{n^2}{8} g^2 k_2) = \frac{1}{2} (k_2 + \frac{n^2}{8} g^2 (\frac{1}{b} - 1) \frac{3n^2 l^4}{8\pi k})$$

$$- g \langle \theta^2 \rangle = - \frac{1}{b^2} (g - \frac{1}{2} g \langle \theta^2 \rangle) = - \frac{1}{b^2} (g + \frac{1}{2} \frac{n^2}{2\pi k} \ln b)$$

$$b = \ln \lambda \quad \frac{d}{d\lambda}$$

$$k = k + \frac{n^2}{2} g^2 (\frac{\lambda - l}{l}) \frac{3n^2 l^4}{8\pi k}$$

$$\frac{dk}{d\lambda} = \frac{dk}{db} \frac{db}{d\lambda} \Rightarrow b \lambda \left[\frac{n^2}{2} g^2 - \frac{1}{l} \frac{3n^2 l^4}{8\pi k} \right] = \frac{3n^2 \lambda^2 g^2}{16\pi k}$$

$$g = \frac{\lambda}{l} (g + \frac{1}{2} \frac{n^2}{2\pi k} \ln \frac{l}{\lambda} g)$$

$$\frac{dg}{db} = \frac{d}{d\lambda} \left[\frac{2\lambda}{l} (g + \frac{1}{2} \frac{n^2}{2\pi k} \ln \frac{l}{\lambda} g) \right]$$

$$+ \frac{\lambda}{l} \left(\frac{1}{2} \frac{n^2}{2\pi k} \frac{1}{\lambda} g \right) \Big|_{\lambda=l} = \left\{ \frac{2}{l} (g + \frac{n^2}{4\pi k} g) - \frac{n^2}{4\pi k} g \right\} = - \frac{n^2}{4\pi k} g$$

$$S = \int dx \frac{k^2}{2} (\partial \theta_c)^2 - g \cos(n \theta_c)$$

$$k_2 = k + \frac{n^2}{8} g^2 k_2$$

$$g_2 = g - \frac{1}{2} K(\omega)$$

$$K(\omega) = \left(\frac{1}{\sqrt{\pi}}\right)^2 \int_{-\frac{2\pi}{\lambda}}^{\frac{2\pi}{\lambda}} \frac{n^2}{|k|k'|^2} = \frac{n^2}{2\pi k} \ln \frac{\lambda}{l}$$

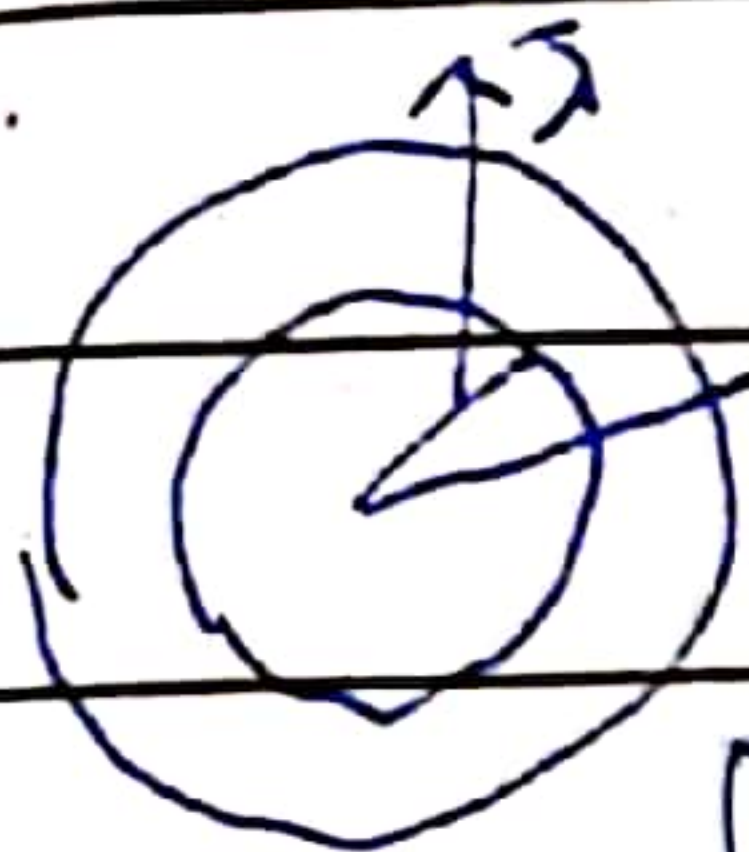
2d BKT fixed point - $\lambda = 1$

$$K_2 = \int dx K(x) \sim \langle \theta(x) \theta(x) \rangle \sim \sum_{k \neq 0} \frac{n^2 e^{i k x}}{|k| k'^2} \sim \frac{\lambda-1}{\epsilon} \frac{3n^2 l^4}{2\pi^2 k}$$

rescaling

$$\int dx \frac{k^2}{2} (\partial \theta_c)^2 - g \cos(n \theta_c) \quad |k| \leq \frac{2\pi}{l} \rightarrow |k| \leq \frac{2\pi}{\lambda} \quad \lambda > 1$$

$$b \cdot \frac{2\pi}{l} = \frac{2\pi}{\lambda} \quad b = \frac{1}{\lambda} \quad |k| \leq b \Lambda \quad k = k k' \quad |k'| \leq \Lambda$$



$$x = \frac{1}{b} x'$$

$$\theta_c(x) = \theta_c\left(\frac{1}{b} x'\right) = \lambda \theta_c(x')$$

$$\int dx \frac{k^2}{2} (\partial \theta_c)^2 - g \cos(n \theta_c)$$

$$= \frac{k\lambda}{2} \int \left(\frac{1}{b}\partial^2 dx'\right) \left(\frac{\partial}{\partial(\frac{1}{b}x)} \theta_c(x)\right)^2 \quad \lambda = 1$$

$$= \frac{k\lambda}{2} \int dx' \left(\frac{\partial}{\partial x'} \theta_c\left(\frac{1}{b}x'\right)\right)^2$$

$$- g \left(\frac{1}{b}\right)^2 \int dx' \cos(n \theta_c(x)) = - g \lambda \left(\frac{1}{b}\right)^2 \int dx' \cos(n \theta_c(x'))$$

$$\text{resulting} = \frac{k\lambda}{2} \int dx' (\partial' \theta_c(x'))^2 - g \lambda \left(\frac{1}{b}\right)^2 \cos(n \theta_c(x'))$$

$$= \frac{1}{2} \left(k + \frac{n^2}{8} g^2 k_2\right) \int dx' (\partial' \theta_c(x'))^2 - \frac{1}{b^2} \left(g - \frac{1}{2} K(\omega)\right) \int dx' \cos(n \theta_c(x'))$$

$(\frac{1}{2} b = \frac{1}{\lambda}) \quad \downarrow \quad k_2 \quad \quad \quad \downarrow \quad g(b)$

$$b = \frac{1}{\lambda} = 1 - \epsilon$$

$$\frac{k}{2} (1 - \epsilon) = \frac{1}{2} \left(k + \frac{n^2}{8} g^2 k_2\right)$$

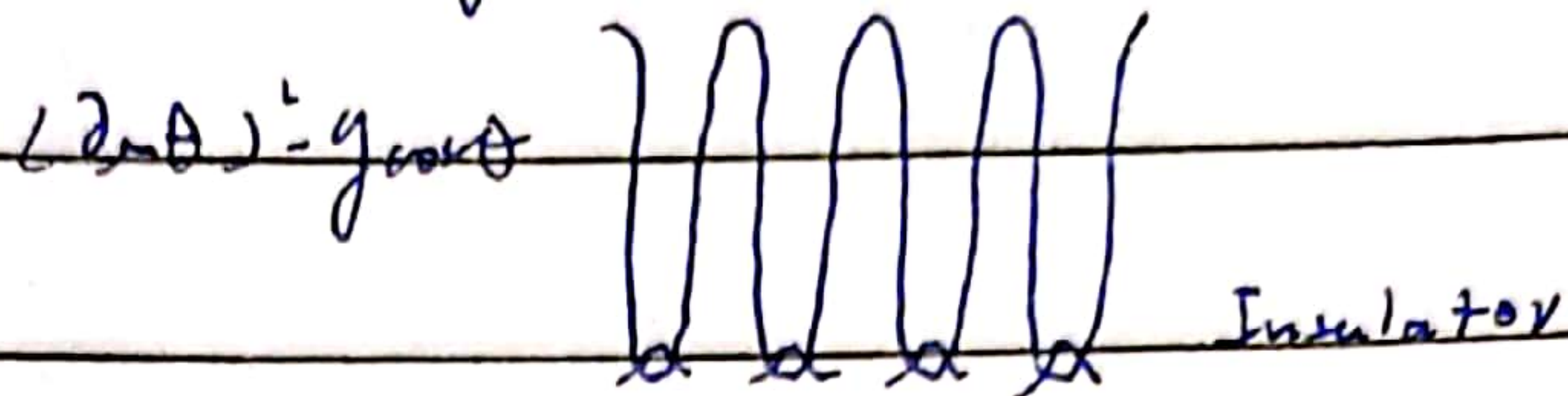
$$- g(1 - \epsilon) = - \frac{1}{b^2} \left(g - \frac{1}{2} K(\omega)\right)$$

$$\begin{cases} \frac{\partial g}{\partial b} = - \frac{n^2 g}{4\pi\lambda} \\ \frac{\partial k}{\partial b} = \frac{3n^4 g^2 \lambda}{16\pi^2 k} \end{cases}$$

initially. $\{ k \rightarrow \bar{k}$
 $g \rightarrow \bar{g}, \lambda^2$

$$\begin{cases} \frac{d\bar{g}}{d\bar{k}} = (2 - \frac{n^2}{4\pi\bar{k}}) \bar{g} \\ \frac{d\bar{k}}{d\bar{g}} = \frac{3n^4 \bar{g}^2}{16\pi^2 \bar{k}} \end{cases}$$

initially. $\textcircled{1} 2 - \frac{n^2}{4\pi\bar{k}} > 0 \quad \bar{g} \rightarrow \infty$ $\rho \rightarrow 0$ insulator



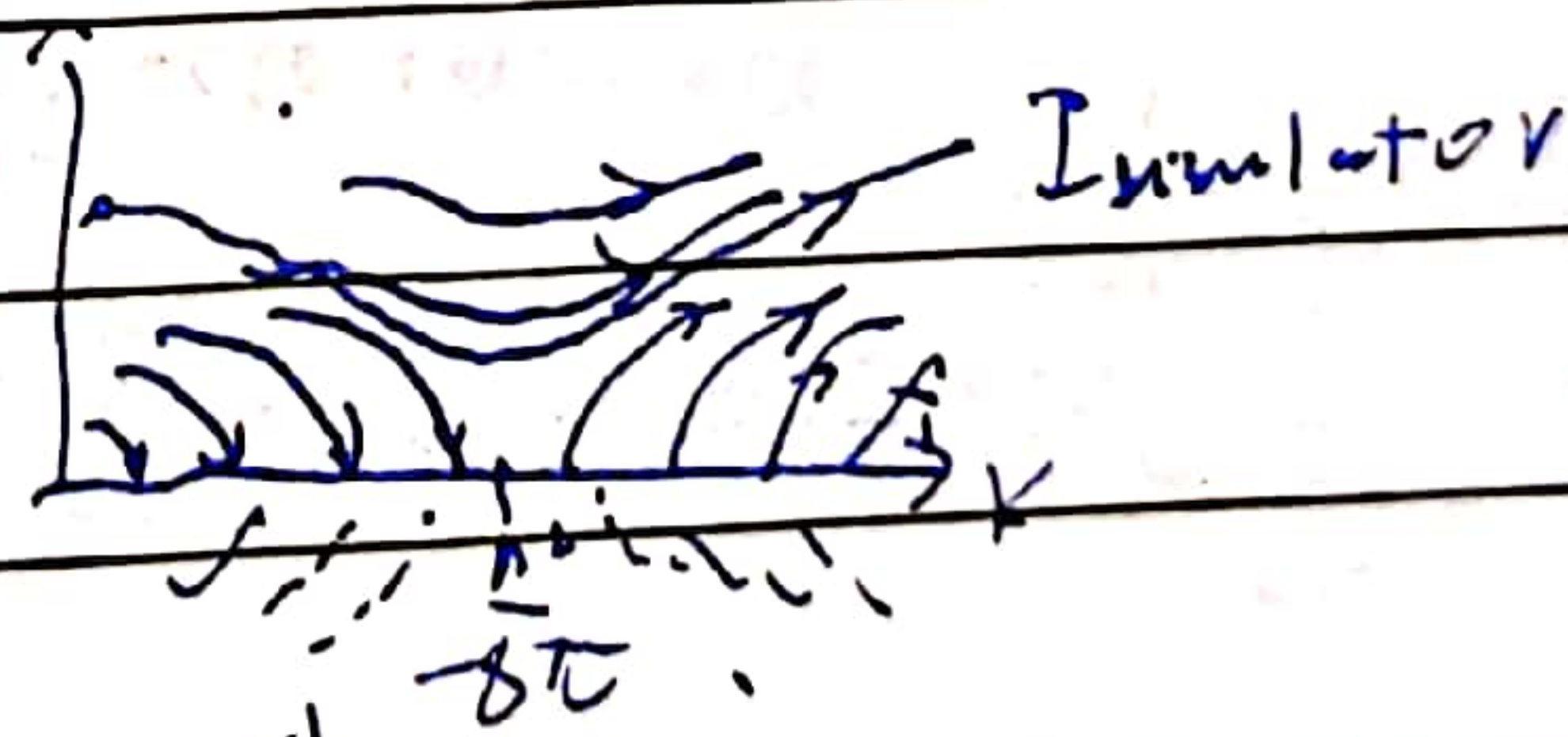
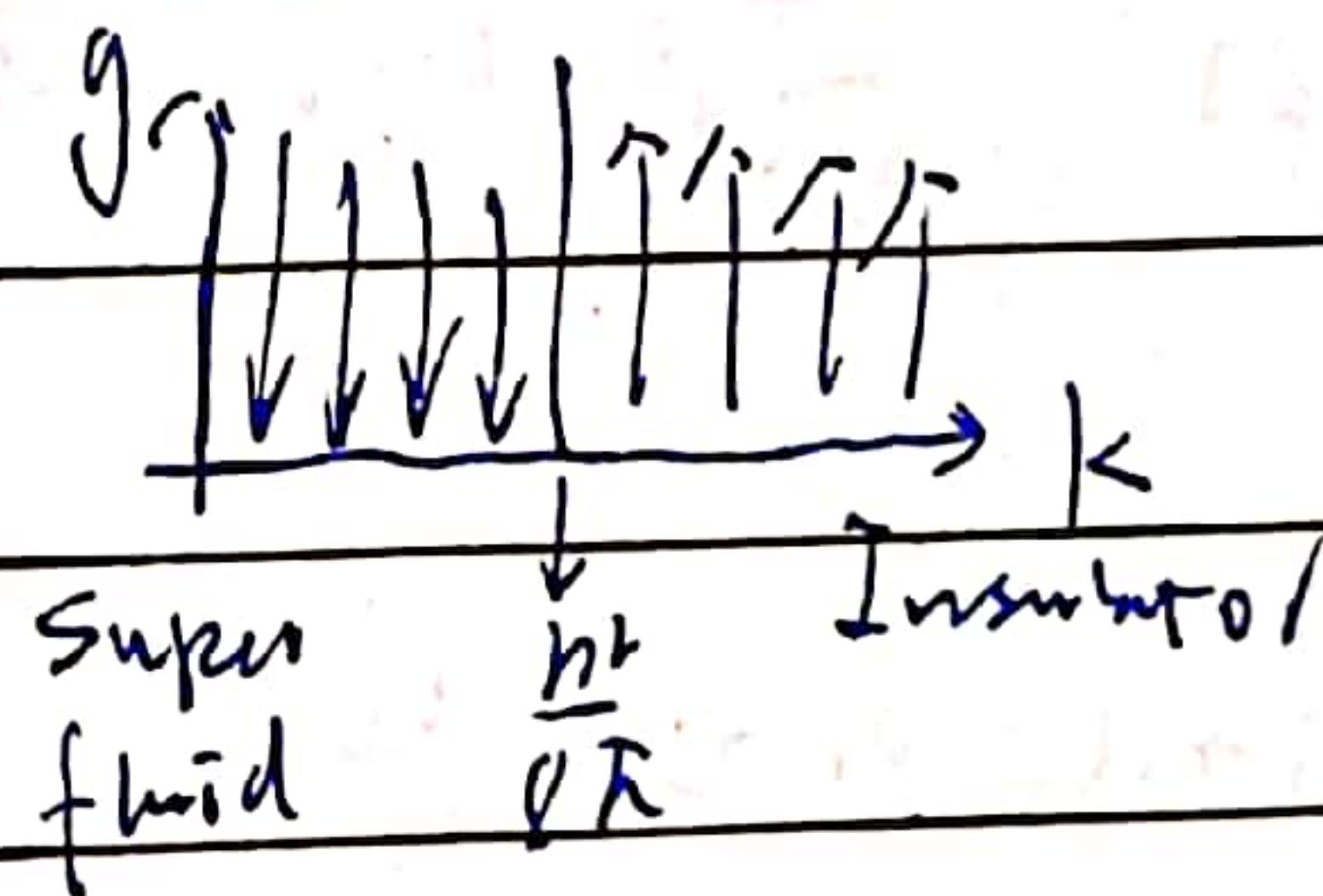
$\textcircled{2} 2 - \frac{n^2}{4\pi\bar{k}} < 0 \quad \bar{g} \rightarrow 0$ $\Rightarrow \int \frac{1}{2} (\partial\theta)^2 \Rightarrow$ superfluid phase

$\textcircled{3} 2 - \frac{n^2}{4\pi\bar{k}} = 0$ fixed point

initially. $T \rightarrow \infty$ $2 - \frac{n^2}{4\pi\bar{k}} = 0$ $K = \beta J$ $\Rightarrow \int \cos(\theta_i - \theta_j) = \cos \frac{1}{2} + \frac{1}{2} J(\partial\theta)^2$
 $e^{-\beta H} \sim e^{\frac{1}{2} \beta J (\partial\theta)^2} \sim e^{\frac{K}{2} (\partial\theta)^2}$ $\beta J = \frac{n^2}{8\pi} = \frac{n^2}{8\pi} \frac{16}{8\pi} = \frac{2}{\pi}$

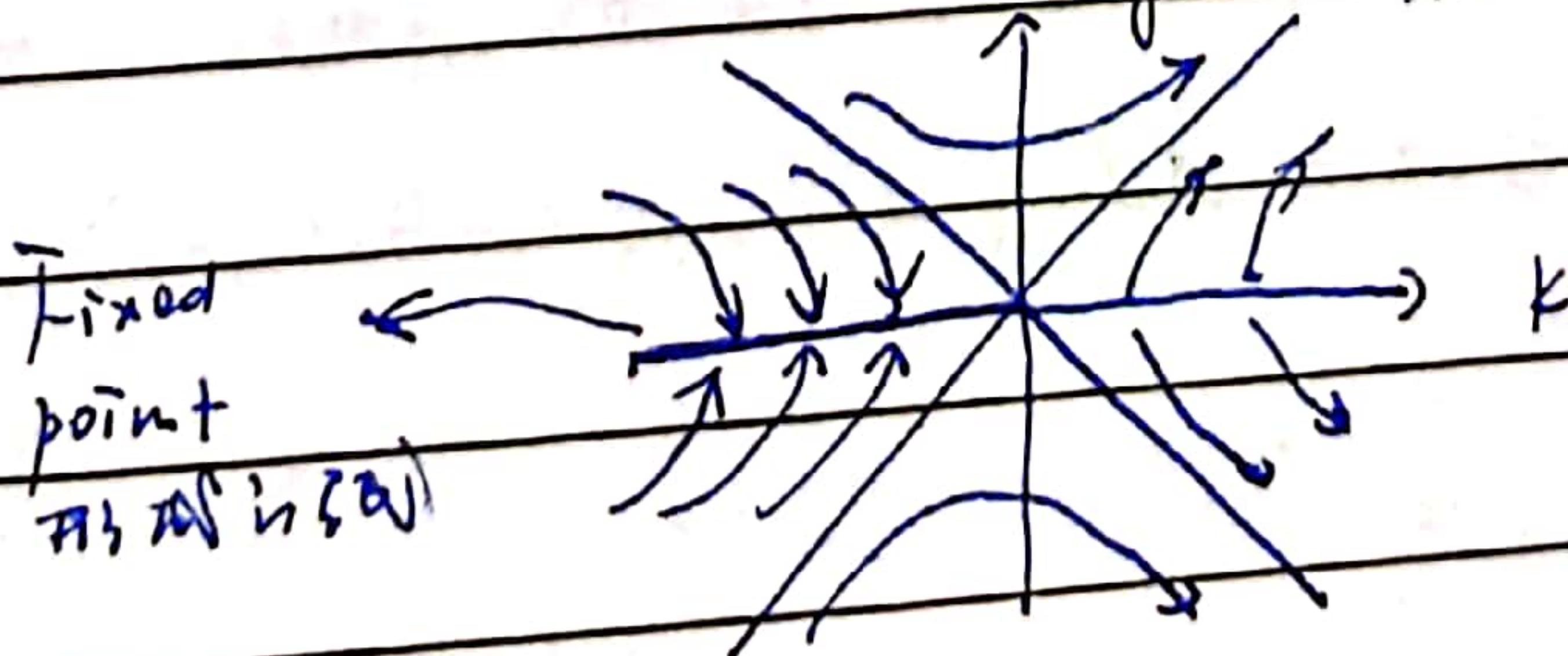
βJ

initially running coupling const.



$\textcircled{4} 2 - \frac{n^2}{4\pi\bar{k}} = k$

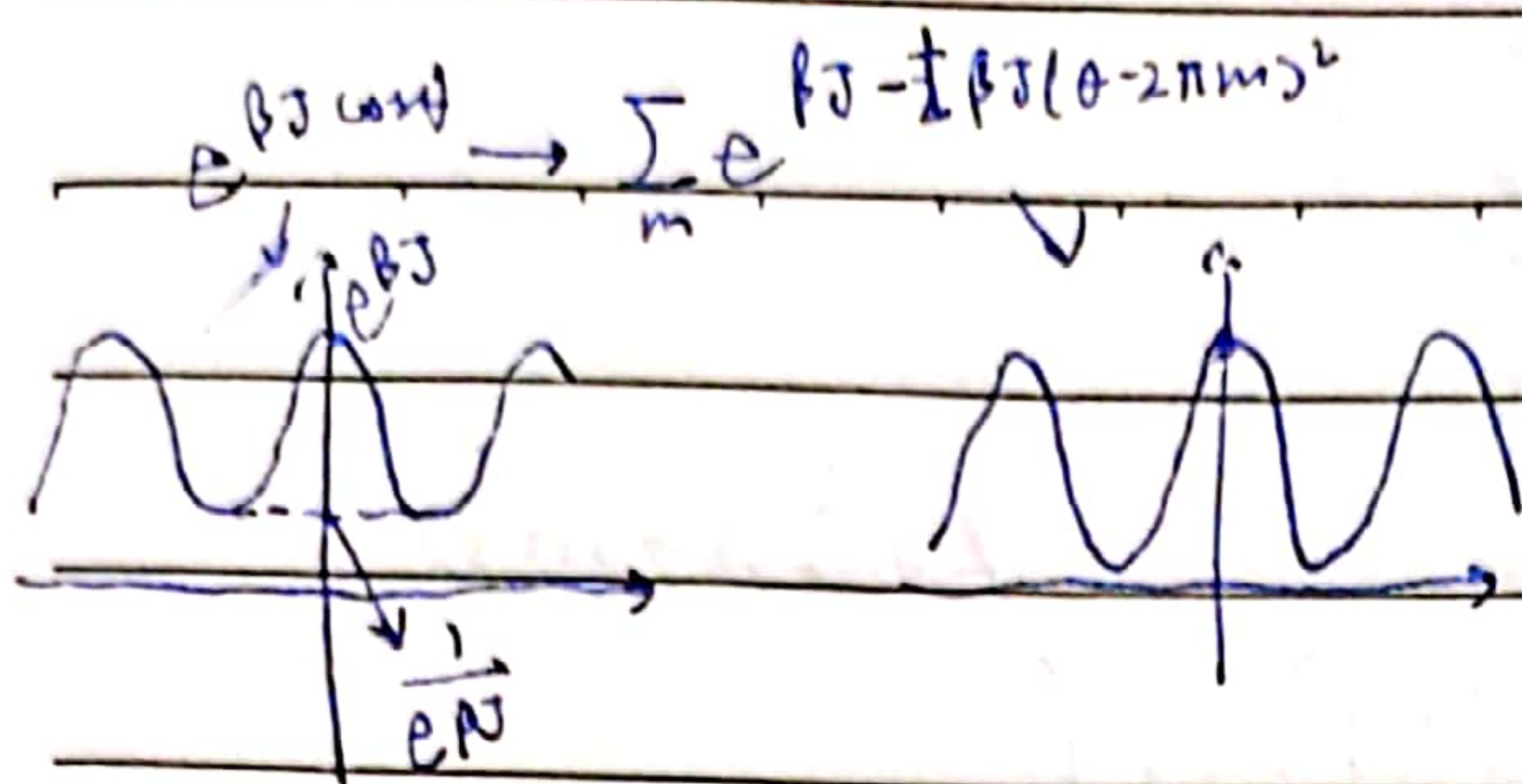
$\begin{cases} \frac{d\bar{g}}{d\bar{k}} = k\bar{g} \\ \frac{d\bar{k}}{d\bar{g}} = \frac{n^2}{4\pi\bar{k}^2} (\frac{d\bar{k}}{d\bar{g}}) \end{cases}$



reference.

Shankar Φ'

Fig. 18.2.



$$Z = \int_{-\infty}^{+\infty} d\theta_1 \dots d\theta_n e^{\beta J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)}$$

$$= \int_{-\infty}^{+\infty} d\theta_1 \dots d\theta_n \prod_{\langle ij \rangle} \sum_m e^{\beta J - \frac{1}{2} \beta J (\theta_i - \theta_j - 2\pi m)^2}$$

$$\sum_{l_{ij}} e^{\beta J - \frac{1}{2} \beta J (\theta_i - \theta_j - 2\pi \phi + 2\pi i l_{ij})^2}$$

$$= \int_{-\infty}^{+\infty} d\theta_1 \dots d\theta_n \prod_{\langle ij \rangle} \int d\phi e^{\beta J - \frac{1}{2} \beta J (\theta_i - \theta_j - 2\pi \phi + 2\pi i l_{ij})^2}$$

$$e^{\beta J} \int d\phi e^{-\left[\frac{\beta J}{2} (\theta_i - \theta_j - 2\pi \phi + 2\pi i l_{ij})^2\right]}$$

$$\frac{1}{\sqrt{2\pi\beta J}} e^{\beta J + i l_{ij} (\theta_i - \theta_j) - \frac{l_{ij}^2}{2\beta J}}$$

$$= \int_{-\infty}^{+\infty} d\theta_1 \dots d\theta_n \prod_{\langle ij \rangle} \frac{1}{\sqrt{2\pi\beta J}} e^{\beta J + i l_{ij} (\theta_i - \theta_j) - \frac{l_{ij}^2}{2\beta J}}$$

$$= \int_{-\infty}^{+\infty} d\theta_1 \dots d\theta_n \prod_{\langle ij \rangle} \frac{1}{\sqrt{2\pi\beta J}} e^{\beta J + \sum_{\langle ij \rangle} i l_{ij} (\theta_i - \theta_j) - \frac{l_{ij}^2}{2\beta J}}$$

$$= \sum_{\{l_{ij}\}} e^{\beta J - \frac{l_{ij}^2}{2\beta J}} \int d\theta_1 \dots d\theta_n e^{\sum_{\langle ij \rangle} i l_{ij} (\theta_i - \theta_j)}$$

$i l_{ij} \theta_i - i l_{ij} \theta_j \rightarrow l_{ij} \rightarrow l_{ij}(\mu)$

$$= \frac{l_{ij}^2}{2\beta J} + \sum_{\langle ij \rangle} i l_{ij} (\theta_i - \theta_j) = -\frac{l_{ij}^2}{2\beta J} + i l_{ij}(\mu) (\theta_{ij} - \theta_{ij+\mu})$$

$$Z = \sum_{\{l_{ij}\}} e^{-\sum_{\langle ij \rangle} \frac{l_{ij}^2}{2\beta J}} \int d\theta_1 \dots d\theta_n e^{\sum_{\langle ij \rangle} i l_{ij}(\mu) (\theta_{ij} - \theta_{ij+\mu})}$$

$$\int d\theta_1 \dots d\theta_n e^{\sum_{\langle ij \rangle} i (l_{ij}(\mu) - l_{ij}(\mu+\mu)) \theta_{ij}}$$

$$= \prod_{\langle ij \rangle} \delta(l_{ij}(\mu) - l_{ij}(\mu+\mu))$$

Wannier-Krammer duality

$$Z = \int_{-\pi}^{+\pi} D\theta_1 \dots D\theta_N e^{\beta J \sum_{i,j} \omega(\theta_i - \theta_j)}$$

for $N=1$ $\theta_1 = \theta$ is θ .

$$e^{\beta J \omega(\theta)} \rightarrow \sum_m e^{\beta J - \frac{1}{2} \beta J (\theta - 2\pi m)^2}$$

for $\theta \in [0, 2\pi)$.

$$\beta J = \frac{2}{\pi} \approx 1.$$

$$\sum_m h_{lm} = \int_{-\pi}^{+\pi} d\phi \tilde{h}(\phi) e^{2\pi i l \phi} = \int_{-\pi}^{+\pi} d\phi \tilde{h}(\phi) \delta(\phi - m)$$

is h_{lm} is h_{lm} .

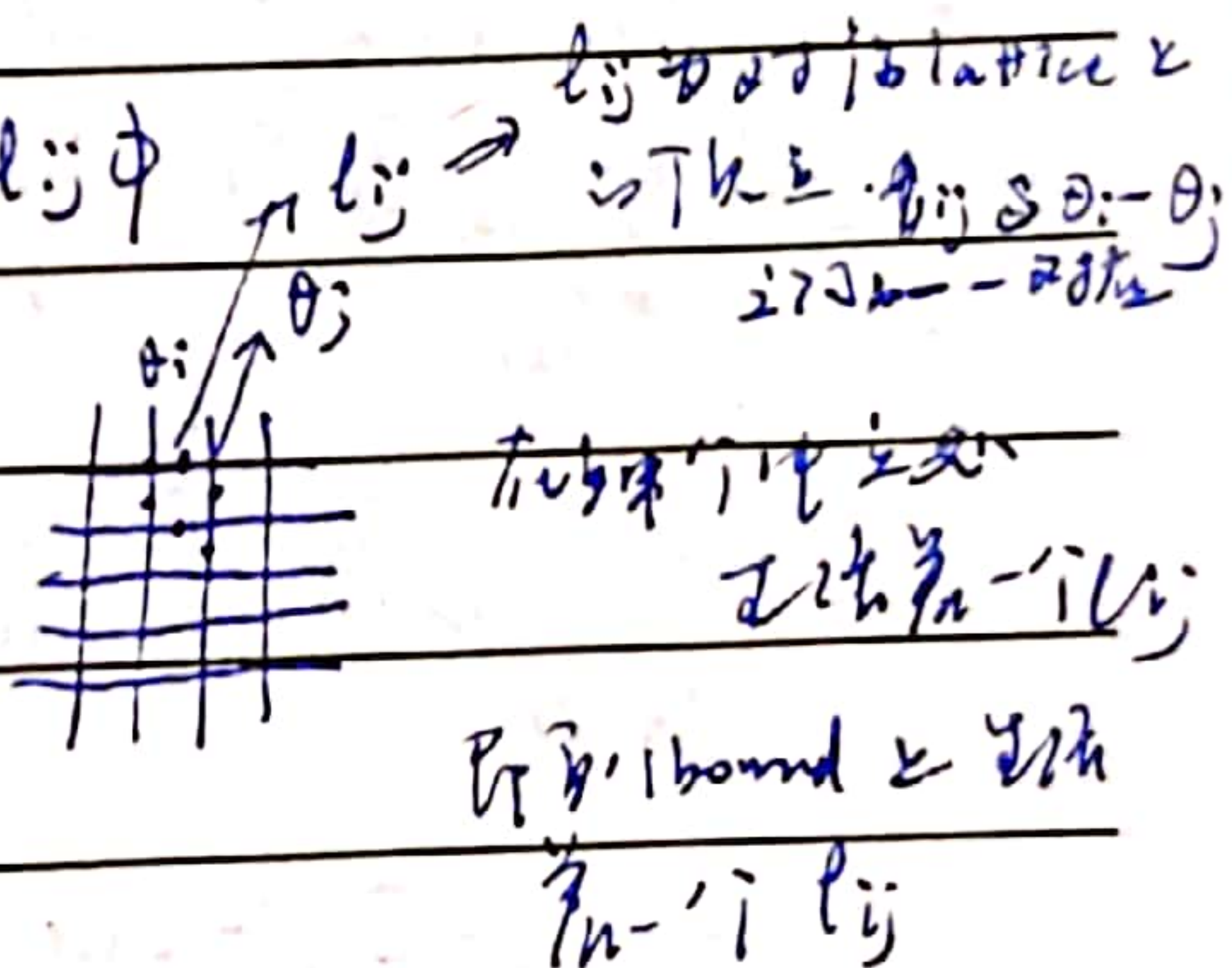
$$Z = \int_{-\pi}^{+\pi} D\theta_1 \dots D\theta_N e^{\beta J \sum_{i,j} \omega(\theta_i - \theta_j)}$$

$$Z = \int D\theta \exp \left[-\frac{\beta J}{2} (\theta_i - \theta_j - 2\pi \phi)^2 + 2\pi i l_{ij} \phi \right] d\phi$$

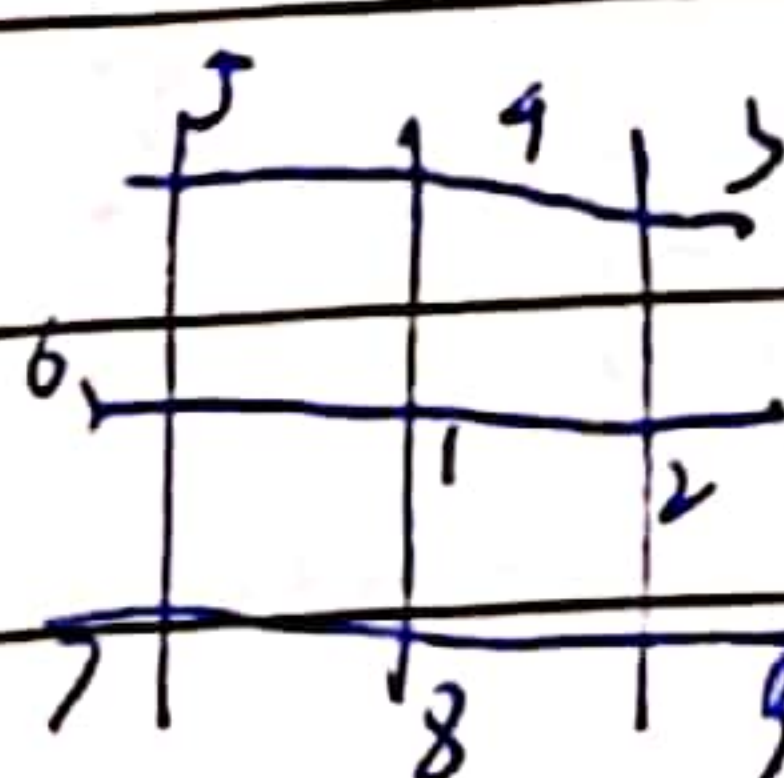
$$= \int D\theta \sum_m e^{\beta J - \frac{1}{2} \beta J (\theta_i - \theta_j - 2\pi m)^2}$$

$$= \int D\theta \sum_{l_{ij}=-\infty}^{+\infty} \int d\phi e^{\beta J - \frac{1}{2} \beta J (\theta_i - \theta_j - 2\pi \phi)^2 + 2\pi i l_{ij} \phi}$$

$$= \int D\theta \frac{1}{\sqrt{2\pi\beta J}} \sum_{l_{ij}} e^{\beta J + i l_{ij} (\theta_i - \theta_j) - \frac{1}{2\beta J} l_{ij}^2}$$



$$Z = \sum_{l_{ij}} e^{-\sum_{i,j} \frac{l_{ij}^2}{2\beta J}} \int D\theta e^{i l_{ij} (\theta_i - \theta_j)}$$



$$l_{ij} = -l_{ji}$$

$$\sum_{i,j} \frac{l_{ij}^2}{2\beta J} = \sum_{i,j} \frac{l_{ij}^2}{2\beta J} = \sum_{i,j} \frac{l_{ij}^2}{2\beta J}$$

$$Z = \sum_{l_{ij}} e^{-\sum_{i,j} \frac{l_{ij}^2}{2\beta J}} \int D\theta e^{i \sum_{i,j} (l_{ij} \theta_i - l_{ij} \theta_j)}$$

YCP

No

Date

$$\vec{L} = \vec{r} \times \vec{p} = (L_x(\vec{r}), L_y(\vec{r})) = \vec{r} \times \vec{p}$$

$$\vec{n} = (0, 0, 1)$$

$$L_x(\vec{r}) = y p_z - z p_y = \partial_y \psi \quad L_y(\vec{r}) = z p_x - x p_z = -\partial_x \psi$$

$$L_x = n(r) - n(r-y)$$

$$L_y(\vec{r}) = n(r-x) - n(r)$$

$$\text{so we } L_x(\vec{r}) - L_x(\vec{r}-x) + L_y(\vec{r}) - L_y(\vec{r}-y) = n(r) - n(r-y) - n(r-x) + n(r-x-y)$$

$$= n(r) + n(r-x) + n(r-y) - n(r-x-y)$$

$$= 0 = \vec{r} \cdot \vec{L}$$

$$Z = \int D\theta_1 \dots D\theta_m e^{i \sum_{i,j} \cos(\theta_i - \theta_j)}$$

$$= \sum_{\{n_{ij}\}} e^{-\sum_{i,j} \frac{L_{ij}^2}{2\beta J}} \prod_r \delta(\sum_{ij} L_{ij}(\vec{r}) - L_{ij}(\vec{r}-\mu))$$

$$= \sum_{\{n_{ij}\}} e^{-\sum_{i,j} \frac{L_{ij}^2}{2\beta J}} \prod_r \delta(\sum_{ij} (L_{ij}(\vec{r}) - L_{ij}(\vec{r}-\mu)))$$

$$\text{for } \vec{r} \in \mathbb{Z}^2 \text{ and } \mu \in \mathbb{Z}^2$$

$$= \sum_{\{n_{ij}\}} e^{-\sum_{i,j} \frac{1}{2\beta J} (n_{ij}(\vec{r}) - n_{ij}(\vec{r}-\mu))^2}$$

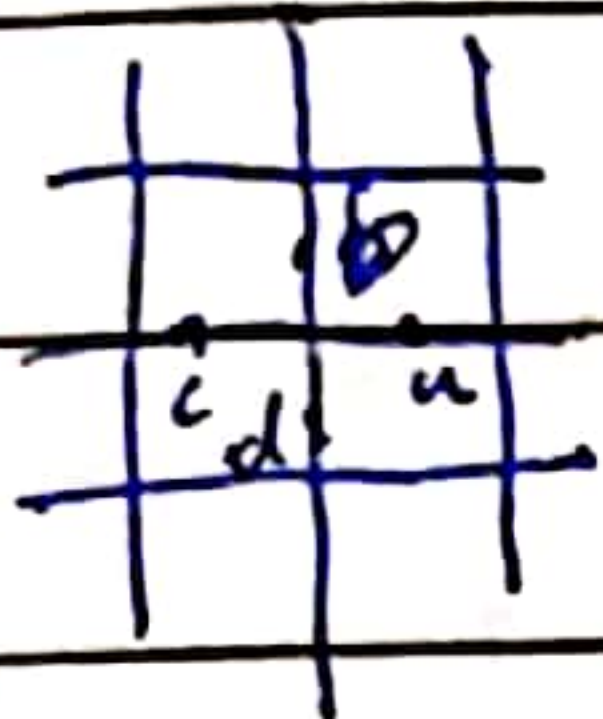
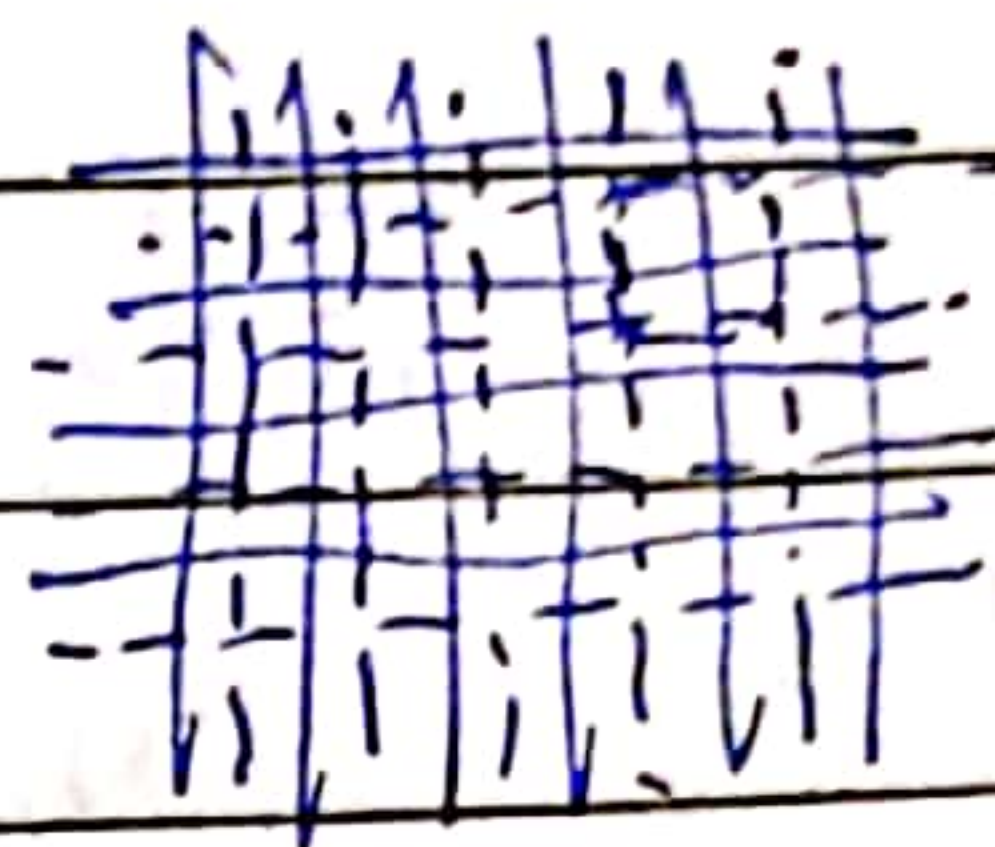
$$\text{BKT: } e^{K \cos(\theta)} = \frac{1}{\sqrt{2\pi K}} \int e^{K - \frac{q^2}{2K} + i q \theta} = \frac{1}{\sqrt{2\pi K}} \int e^{K - \frac{q^2}{2K} + i q \theta}$$

2nd: Villain transformation + Poisson sum

$$\delta \left[\sum_{\mathbf{r}} (l_{\mathbf{r}}(r) - l_{\mathbf{r}}(r-\mu)) \right]$$

$$i \sum_{\mathbf{r}} \theta_{\mathbf{r}} \sum_{\mu} (l_{\mathbf{r}}(r) - l_{\mathbf{r}}(r-\mu))$$

$$\int e^{-\frac{1}{2\beta J} \sum_{\mathbf{r}} l_{\mathbf{r}}^2} \prod_{\mathbf{r}} d\theta_{\mathbf{r}} e^{\sum_{\mathbf{r}} i l_{\mathbf{r}}(r) (\theta_{\mathbf{r}} - \theta_{\mathbf{r}+\mu})} \delta \left[\sum_{\mathbf{r}} (l_{\mathbf{r}}(r) - l_{\mathbf{r}}(r-\mu)) \right]$$



$$a - c + b - d = 0$$

for $\vec{l}(r) = (l_x(r), l_y(r)) = (\nabla \times \vec{n})$

$$l_x = \partial_y n \quad l_y = -\partial_x n$$

$$\vec{n} = (0, 0, n) \parallel \hat{z}$$

$$l_x = n(r) - n(r-y) \quad l_y = -n(r) + n(r-x)$$

for $\vec{l}(r) = (l_x(r), l_y(r))$ $l_x(r) - l_x(r-x) + l_y(r) - l_y(r-x) = 0$

dual lattice $\times$ $l(r) \in \mathbb{Z}$. $\vec{l} = \nabla \times \vec{n}$ is in $\mathbb{Z}$. $\vec{n} \times \vec{l} = 0$

$$l_x(r) = n(r) - n(r-y)$$

$$l_y(r) = -n(r) + n(r-x)$$

$$\vec{l} \cdot \vec{n} = 0$$

$$Z = \int D\theta e^{i \sum_{\mathbf{r}} \theta_{\mathbf{r}} (l_{\mathbf{r}}(r) - l_{\mathbf{r}}(r-\mu))} = \sum_{\{l_{\mathbf{r}}\}} e^{-\frac{1}{2\beta J} \sum_{\mathbf{r}} l_{\mathbf{r}}^2} \delta \left(\sum_{\mathbf{r}} (l_{\mathbf{r}}(r) - l_{\mathbf{r}}(r-\mu)) \right)$$

$$= \sum_{\{l_{\mathbf{r}}\}} \sum_{\{n(r)\}} e^{-\frac{1}{2\beta J} \sum_{\mathbf{r}} (n(r) - n(r-\mu))^2}$$

or Poisson equation

for $\vec{l}$ on Honeycomb lattice or BKT to be

ref: wagner duality in generalized Ising Model

ref: wagner duality in generalized Ising Model