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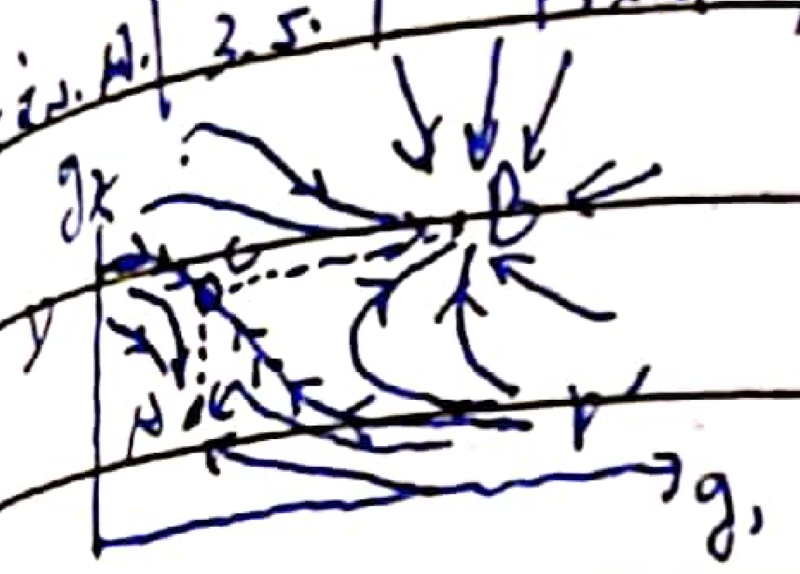
§2.5.7 Fixed Point and Universality >

普适类



① 临界点与普适类

② stable phase ⇒ 对 δH 都是 irrelevant.

③ 相变点 δH ⇒ Flow 到不同的 stable phases

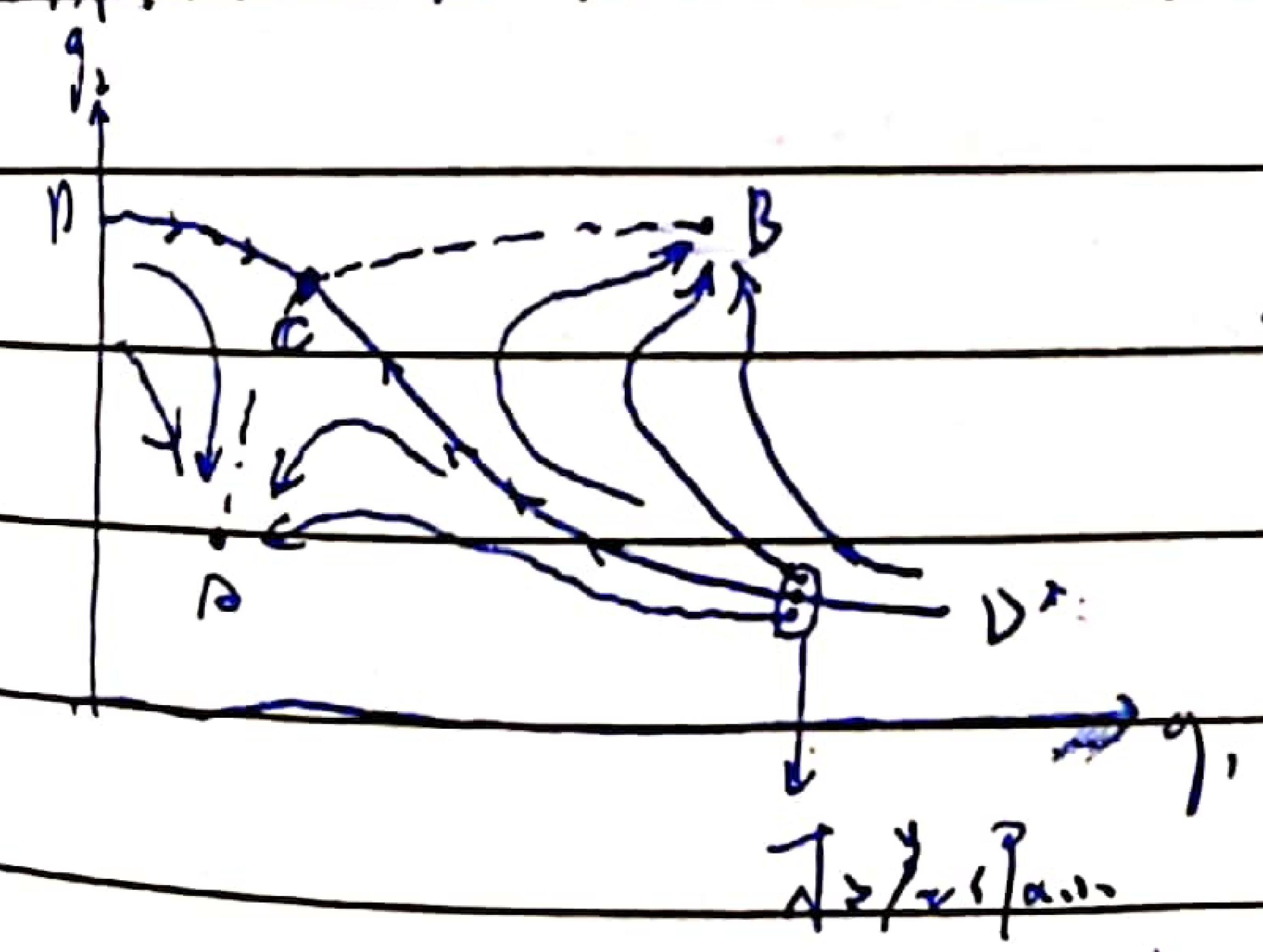
基本 RG: → $\Lambda \to b\Lambda \to b^2\Lambda \cdots\to$ <Λ的 ⇒ 平均场

stable: 平均场 + fluctuation.
高阶

Flow: $\Lambda\to b\Lambda\to b^2\Lambda\to\cdots\sim 0$ $\quad g\to g_{b\Lambda}\to g_{b^2\Lambda}\to g_{b^3\Lambda}\to\cdots\cdots\to g_0$

stable Fixed Point

不同的系统 L or H(amiltonian) 但一般在相变点附近 flow 到同一个地方



相线 DD' 区分了两个不同的 phase. 即 DD' 均为相变点

相变点附近，它们…… 临界点都会导致走向不同的 phase

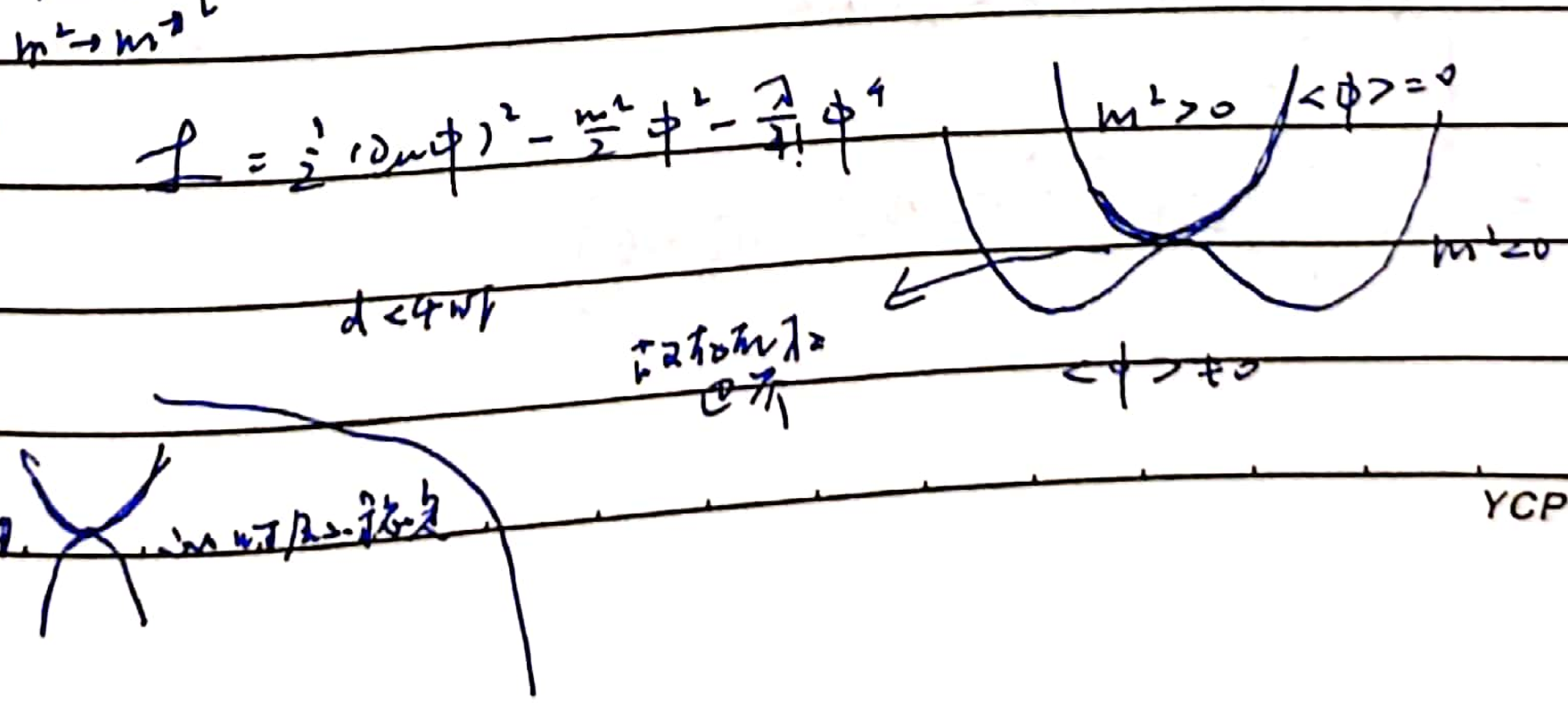
$$\frac{d}{dl}\begin{pmatrix}\delta\lambda\\ \delta m^2\end{pmatrix} = A^*\begin{pmatrix}\delta\lambda\\ \delta m^2\end{pmatrix}$$

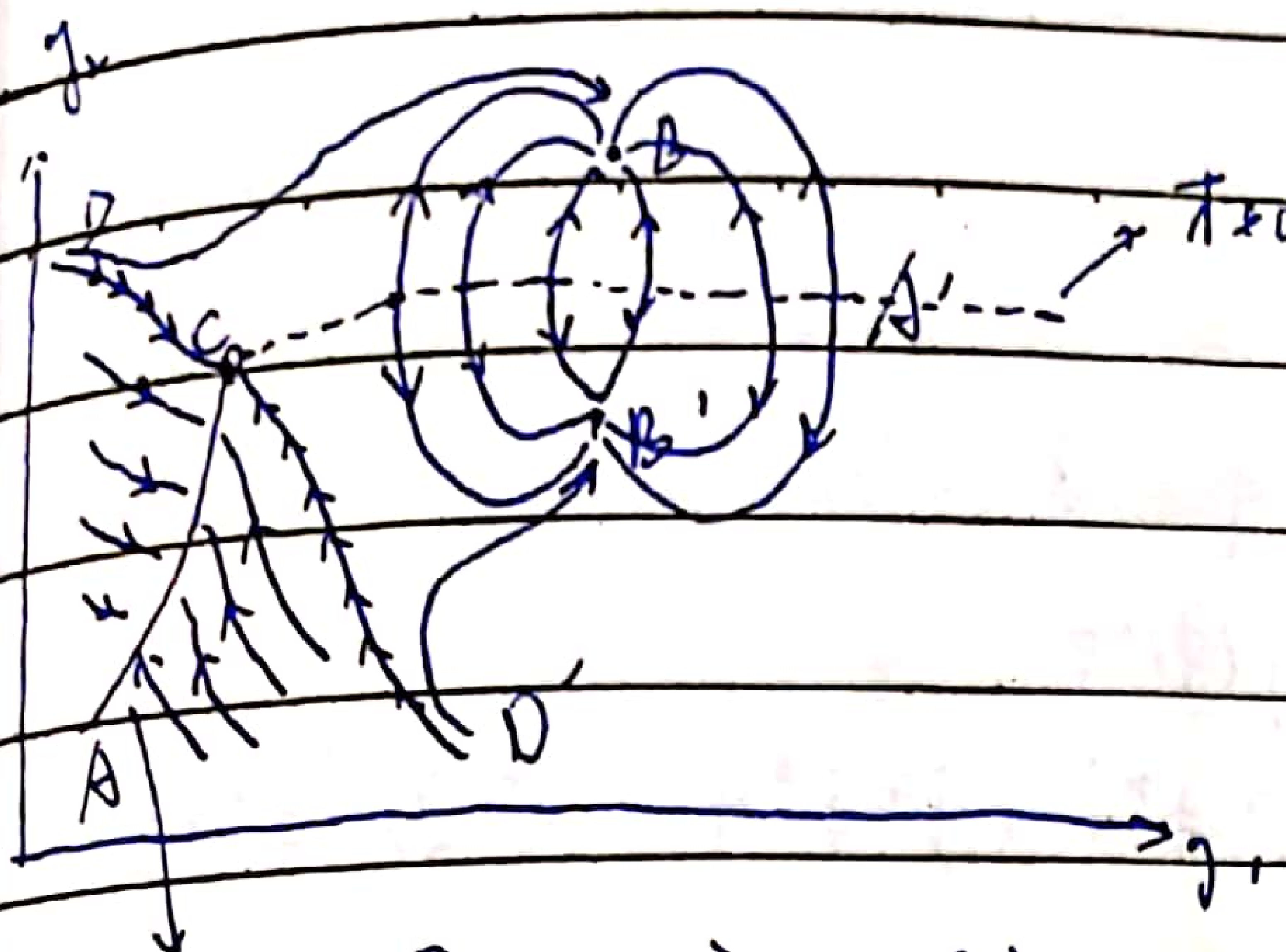
$\frac{d\vec{x}}{dt} = A\vec{x}$ 这样的线性化在不动点附近 $\lambda\to\lambda^*$, $m^2\to m^{*2}$

$x=\sum_n x_n e^{\lambda_n t}$

A^* {
所有本征值实部都 <0 $\quad Re(\lambda_n)<0$
$\exists Re(\lambda_n)>0$
$\exists Re(\lambda_n)=0$ (称 Marginal point 临界点)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$





AC 线上的点都流向 A，AC 线是临界线

stable fixed line (low energy physics 是 mean field theory)

J_2 D CD' 亦为相变线　　T_c 决定 { ① fixed point universality → g_2^* 决定了临界指数
　　　　　　　　　　　　　　　　　　 ② 相变线

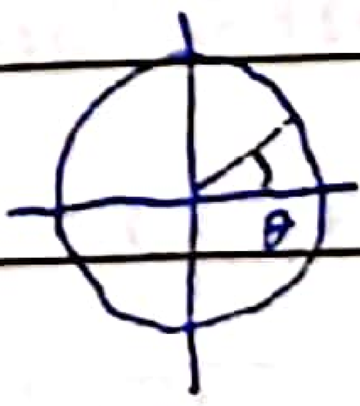
BKT相变.

60-70s. Mermin-Wagner 1966 $d=2$ 低温时无长程有序　　1969-1971. Wilson ⇒ R.G.　1971 ⇒ Thouless ⇒ BKT相变

二维相变 1. 2d Ising Model　2. 3d Ising Model　3. 1d 相变不稳定　4. 2d 低温也不稳定 → Thouless
XY、XXZ Model 相变　　5. 1961. 数值结果
1967. Hohenberg. $d=1$, $d=2$ 不存在长程有序

BKT相变

1. 模型



$\vec{I}_i = (I_i^x, I_i^y)$

{ 低温时 ⇒ Nagaosa 书上
BEC.

用：模型不同 但 相变一样

$$H = -J\sum_{ij}\vec{I}_i\cdot\vec{I}_j - \sum_i \vec{h}_i\cdot\vec{I}_i$$

$$= -J\sum_{ij} I^2\cos(\theta_i-\theta_j) - h\sum_i \cos\theta_i$$

$$-JI^2\cos(a\nabla\theta) - hI\cos\theta$$

$$= -JI^2 + \frac{J}{2}I^2a^2(\nabla\theta)^2 - hI\cos\theta$$

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$H = \phi^*\left(\frac{p^2}{2m}-\mu\right)\phi + \frac{g}{2}(\phi^*\phi)^2$

$\mathcal{L} = i\hbar\phi^*\dot\phi - H = i\hbar\phi^*\dot\phi - \phi^*\left(\frac{p^2}{2m}-\mu\right)\phi - \frac{g}{2}(\phi^*\phi)^2$

$\frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial\dot\phi^*} = 0 = \frac{\partial \mathcal{L}}{\partial\phi^*} = i\hbar\dot\phi - \left(\frac{p^2}{2m}-\mu\right)\phi - g|\phi|^2\phi$

$\mathcal{L}' = \frac{i\hbar}{2}(\phi^*\dot\phi - \dot\phi^*\phi) - H$ $\quad L-L' = \frac{i\hbar}{2}\phi^*\dot\phi + \frac{i\hbar}{2}\dot\phi^*\phi = \frac{i\hbar}{2}\frac{\partial}{\partial t}(\phi^*\phi)$

故L与L'等价.

$\mathcal{L} = \overset{①}{\frac{i}{2}(\phi^*\dot\phi - \dot\phi^*\phi)} - \overset{②}{\frac{1}{2m}(\nabla\phi^*)(\nabla\phi)} + \overset{③}{\mu|\phi|^2} - \overset{④}{\frac{V}{2}(|\phi|^2)^2}$

$\phi = \sqrt{\rho+\delta\rho}\,e^{i\theta}$

③: $\mu(\rho+\delta\rho)$ (变分时,由于 $\int\delta\rho = 0$ 则这一项给出 $\mu\rho$

④ $-\frac{V}{2}(\rho^2+\delta\rho^2+2\delta\rho\,\rho)$ $\quad \int\delta\rho=0 \Rightarrow -\frac{V}{2}\int\rho^2+\delta\rho^2$

计算①-②.

① $\frac{i}{2}\left\{\sqrt{\rho+\delta\rho}\,e^{-i\theta}\sqrt{\rho+\delta\rho}\,e^{i\theta}\,i\partial_t\theta + \frac{1}{2}\partial_t\delta\rho - \sqrt{\rho+\delta\rho}\,e^{i\theta}\sqrt{\rho+\delta\rho}\,(-i)e^{-i\theta}\partial_t\theta - \frac{1}{2}\partial_t\delta\rho\right\}$

$= -(\rho+\delta\rho)\partial_t\theta$

② $\frac{1}{2m}\left\{\sqrt{\rho+\delta\rho}\,e^{-i\theta}(-i)\nabla\theta + \frac{1}{2}\frac{\nabla\delta\rho}{\sqrt{\rho+\delta\rho}}e^{-i\theta}\right\}\left\{\sqrt{\rho+\delta\rho}\,e^{i\theta}\,i\nabla\theta + \frac{\nabla\delta\rho}{2\sqrt{\rho+\delta\rho}}e^{i\theta}\right\}$

$= \frac{\rho+\delta\rho}{2m}(\nabla\theta)^2 + \frac{(\nabla\delta\rho)^2}{8m(\rho+\delta\rho)} + \frac{1}{4m}\nabla\delta\rho\, i\nabla\theta - \frac{1}{4m}\nabla\delta\rho\, i\nabla\theta$

$= \frac{\rho+\delta\rho}{2m}(\nabla\theta)^2 + \frac{(\nabla\delta\rho)^2}{8m(\rho+\delta\rho)}$

得到 $\mathcal{L}_{eff} = -(\rho+\delta\rho)\partial_t\theta - \frac{\rho}{2m}(\nabla\theta)^2 - \frac{(\nabla\delta\rho)^2}{8m\rho} - \frac{V}{2}\delta\rho^2 + \mathcal{L}(\rho)$

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文小刚书第三章

相互作用BEC与超流 (3.3节)

a. Schrödinger Equation 和 [illegible]

$$i\frac{\partial}{\partial t}\phi = \left(\frac{p^2}{2m}-\mu\right)\phi + g|\phi|^2\phi \leftarrow \text{Interaction}$$

Boson field

$\Rightarrow \begin{cases} L=? \\ H=? \end{cases}$ $H\phi = i\partial_t\phi$ $\frac{\partial L}{\partial\phi^*} = \frac{d}{dt}\frac{\partial L}{\partial\dot\phi^*} = 0$

$$H = \phi^+\left(\frac{p^2}{2m}-\mu\right)\phi + \frac{g}{2}(\phi^+\phi)^2$$

$L = i\hbar\phi^*\dot\phi - H$ ⇒ [illegible]

$L' = \frac{i\hbar}{2}(\phi^*\dot\phi - \dot\phi^*\phi) - H$ ⇒ [illegible]

L、L' 等价 $L - L' = \frac{i\hbar}{2}(\phi^*\dot\phi + \phi\dot\phi^*) = \frac{i\hbar}{2}\frac{\partial}{\partial t}(\phi^*\phi)$

$\pi = \frac{\partial L}{\partial\dot\phi} = i\hbar\phi^*$ $\pi^* = -i\hbar\phi \neq \frac{\partial L}{\partial\dot\phi^*} = 0$

$L \to L + \frac{d}{dt}f(\phi)$ 可下 Lagrangian 不是一一对应 [illegible]

(constraint quantization)

$L = \frac{i\hbar}{2}(\phi^*\dot\phi - \phi\dot\phi^*) - H$

$Z = \int D\phi D\phi^* e^{iS}$ $S = \int d^dx\, dt\, L$. [illegible] $\phi \to \bar\phi + \delta\phi$

采用 $\phi = \sqrt{\rho+\delta\rho}\, e^{i\theta}$ 是 $\delta\rho$ 涨落 [illegible] $\delta\theta$ 涨落 [illegible]

[illegible] $(\partial_t\delta\phi)^2 - v^2(\partial_x\delta\phi)^2 - \underset{\Downarrow}{g\,\delta\phi^2}$ E_g

$E^2 = v^2k^2 + g \longrightarrow$ gap [illegible] $T \ll$ energy gap

[illegible] $\Longleftrightarrow (\partial_t\theta)^2 - v^2(\partial_x\theta)^2$ [illegible] θ^2 [illegible] $T \ll E_g$

$\theta + \theta_0$ [illegible]

[illegible] $\phi = v e^{i\phi}$

$L = \frac{i}{2}(\phi^*\dot\phi - \phi\dot\phi^*) - \frac{1}{2m}(\partial\phi^*)(\partial\phi) + \mu|\phi|^2 - \frac{V}{2}(|\phi|^2)^2$

$\phi = \sqrt{\rho+\delta\rho}\, e^{i\theta}$

只与 ρ、$\delta\rho$ 有关

$\rho + \delta\rho \Rightarrow \int\delta\rho\, d^dx = 0$ ([illegible])

[illegible] $\rho^2 + \delta\rho^2 + 2\rho\delta\rho$

$-\frac{V_0}{2}(\rho^2 + \delta\rho^2)$

什么相互作用有 gap? 1. [illegible] 2. [illegible]

cooper YCPW CDW

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[illegible] $\delta\rho$ [illegible]

$$\mathcal{L}_{eff} = -\rho\,\partial_t\theta - \frac{\rho}{2m}(\nabla\theta)^2 \qquad \text{则} \quad \int dt\,dx\,\rho\,\partial_t\theta = 0$$

$$\downarrow$$

$$-\frac{\rho}{2m}(\nabla\theta)^2$$

[illegible] $\delta\rho$ [illegible]

$$-(\rho+\delta\rho)\partial_t\theta - \frac{\rho}{2m}(\nabla\theta)^2 - (\nabla\delta\rho)^2/8m\rho - \frac{V}{2}\delta\rho^2$$

$$= -\cancel{\rho\,\partial_t\theta} - \frac{\rho}{2m}(\nabla\theta)^2 - \frac{V}{2}\left(\delta\rho^2 + \frac{2}{V}\delta\rho\,\partial_t\theta + \frac{1}{V^2}(\partial_t\theta)^2 - \frac{1}{V^2}(\partial_t\theta)^2\right)$$

$$\doteq -\frac{\rho}{2m}(\nabla\theta)^2 + \frac{1}{2V}(\partial_t\theta)^2 - \frac{V}{2}\left[\delta\rho + \frac{1}{V}\partial_t\theta\right]^2$$

$\partial_t(\delta\rho)$

$\int \partial_t(\delta\rho) \Rightarrow 0$ 而后消失.（为总电荷守恒）

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$-(\rho+\delta\rho)\partial_t\theta - \frac{\rho}{2m}(\nabla\theta)^2 - \cancel{\frac{(\nabla\delta\rho)^2}{8m\rho}} - \frac{V}{2}\delta\rho^2 + \mathcal{L}_0(\rho)$

低能时看,忽略高 $\delta\rho$. 得到 λ 表达式称之 $\mathcal{L}_{eff}$ （~~高阶~~

$\mathcal{L}_{eff} = -\underset{\downarrow}{\rho\partial_t\theta} - \frac{\rho}{2m}(\nabla\theta)^2 \longrightarrow$ 描述时间演化

$\int dx\,dt\,\rho\partial_t\theta = 0$

考虑 $\delta\rho$ 的贡献, 则有 $(\partial_t\theta)^2 - V^2(\partial_x\theta)^2$ 波动方程

方法: Gaussian Integration. $\int D\phi\, e^{-A\phi^2 + J\phi} = \cancel{\text{const}\, e^{\frac{b^2}{4A}}}$

$\int dx\, e^{-Ax^2+bx} = \text{const}\; e^{\left(\frac{b^2}{4A}\right)}$

$\mathcal{L} = -\delta\rho\,\partial_t\theta - \underline{\frac{\rho}{2m}(\nabla\theta)^2} - \frac{V}{2}\delta\rho^2 - \cancel{\frac{(\nabla\delta\rho)^2}{8m\rho}}$

$k^2\delta\rho^*\delta\rho$ $\varepsilon = \sqrt{\ldots}$ 这个 k^2 很小.但在高能时 当在 $k\to 0$ 处

$= -\frac{V}{2}\left(\underbrace{\delta\rho^2 + \frac{2\partial_t\theta}{V}\delta\rho + \frac{4}{V^2}(\partial_t\theta)^2}_{\text{一个平移.}} - \underline{\frac{4}{V^2}(\partial_t\theta)^2}\right)$

得到 Effective: $S_{eff} = \int dx\,dt\left[\frac{1}{2V}(\partial_t\theta)^2 - \frac{\rho}{2m}(\nabla\theta)^2\right]$ （行波的速度）

$\theta = \theta_k e^{i(\omega t - \vec{k}\cdot\vec{x})}$ $\quad \frac{\omega^2}{2V} = \frac{\rho}{2m}\vec{k}^2 \qquad \omega^2 = \frac{\rho V}{m}\vec{k}^2$

$\omega = \sqrt{\frac{\rho V}{m}\vec{k}^2} = \underset{\text{声速}}{\sqrt{\frac{\rho V}{m}}}\,|\vec{k}|$

$\phi = \sqrt{\rho+\delta\rho}\; e^{i\theta}$ $\qquad \rho$ 是相互作用强度. 但相互作用强度不变

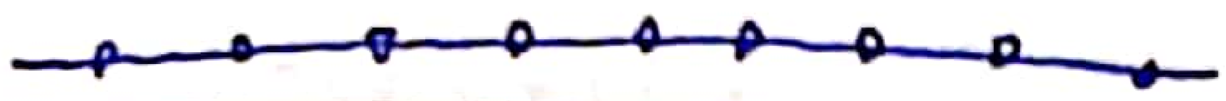
参考 X.G. Wen 书 Problem 3.3.3 P84（习题）

计算关联（计算相互作用下的关联函数）

$\mathcal{L} = \frac{\chi}{2}\left((\partial_t\theta)^2 - V^2(\partial_x\theta)^2\right)$ $\qquad \langle\phi^+(x)\phi(0)\rangle$

$= \rho\int D\theta\, e^{i\theta(x) - i\theta(0) - i\int dx\,dt\,\mathcal{L}}$ $\qquad$（下节课）

以及低维(低维)长程序不稳定



1. $\langle x_i^2 \rangle = \frac{1}{L}\sum_k n_k \sim \int_0^{\Lambda} dk \frac{1}{e^{\beta \gamma k}-1}$ → 发散 ($1 \geq 2$)

1.2 铁磁. $\langle M_i^2 \rangle \sim \frac{1}{V}\sum_k \overset{\text{magnon}}{n_k} \sim \int \frac{1}{e^{\beta v k^2}-1} k\,dk$ 不稳定

$\int \frac{1}{e^{\beta v k}-1} k\,dk$ 可以稳定

2 BEC. $N_{excited} \gg N$. 发散 不稳定

1.3.5. Superfluid as a toy universe 附着 vertex

(光子 → $\frac{1}{r}$ → 电磁力) 守恒 旋转的作用 声子 roton → $\frac{1}{r^4}$ 相互作用 (交换roton/声子)

BKT 相变. $(\partial_t\theta)^2 - v^2(\nabla\theta)^2$ or $(\nabla\theta)^2 + h\cos\theta$

superfluid superconductor XY model.

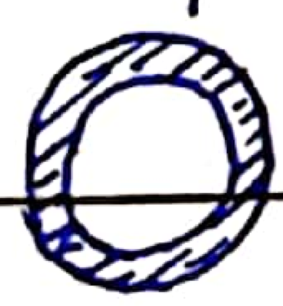
1 有超流的系统 $\sqrt{\rho+\delta\rho}\, e^{i\theta} = \psi$ 加一个非零的涨落. ($\sqrt{\rho+\delta\rho}\, e^{i\delta\theta}$

$\mathcal{L}(\delta\rho, \delta\theta)$ 中 $\partial_t\delta\theta$ $\partial_t\delta\rho$ $\delta\rho\,\partial_t\delta\rho$ 等项不存在

$\delta\rho\,\partial_t\delta\rho$ $(\partial_x\delta\rho)^2$, $(\partial_x\delta\theta)^2$ $\partial_x\delta\rho$ $(\delta\theta)^2$ 不能存在

$U(1)$ → ψ → $\psi e^{i\theta}$ 故 $(\delta\theta)^2$ 不能存在

10 标.



$dy = Dy\,dl$ $\frac{dy}{dl} = D \cdot y$ $y \propto e^{Dl}$

D 阶项. 注意额外的项 $\underset{\Downarrow}{A}\, y\,dl$

修正项

$dy = (D+A)y \cdot dl$ $y \propto e^{(D+A)l}$

相变行为 D+A 决定 D+A 为 0 时变成 relevant 到 irrelevant : BKT相变