参 文献图 3.5.7. Fixed Point and Universality.

1. Fixed Point and Universality.
2. Stable phase → δH 都是 irrelevant.

兩耦合常數 g_1、g_2.

相邊界 → 存在一個 δH, Flow 到不同的 Stable point.

直觀、RG ⟷ ~~$(\Lambda + b\Lambda + b^2\Lambda + \cdots)$~~ → $\Lambda + b^{-1}\Lambda + b^{-2}\Lambda + \cdots$ 低能 → 平均場、

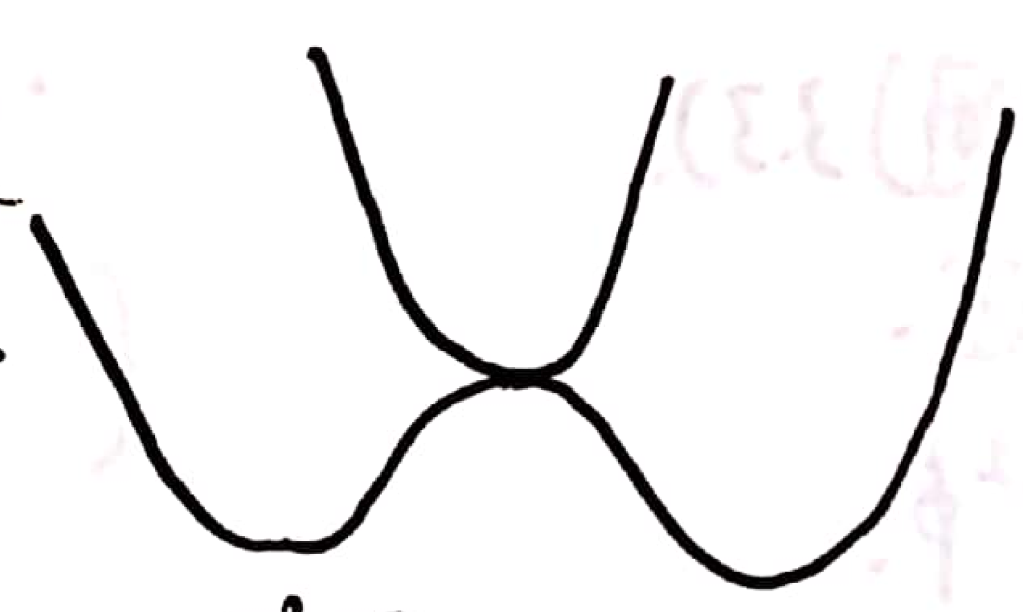
Stable 平均場 + 漲落 ⟺ 高能、(逐漸循環上限

Flow $\Lambda \to b\Lambda \to b^2\Lambda \to \cdots \sim 0$ 低能、

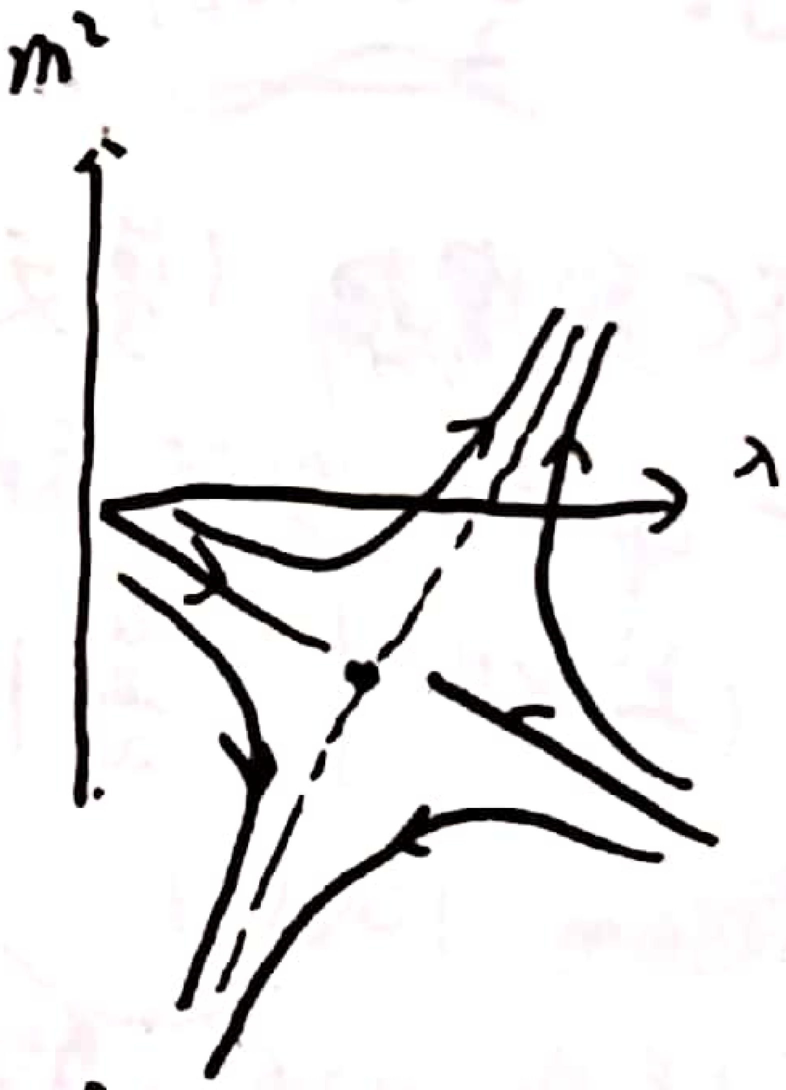
$g_\Lambda \to g_{b\Lambda} \to g_{b^2\Lambda} \to \cdots g_0$ ⟹ g^* fixed point.

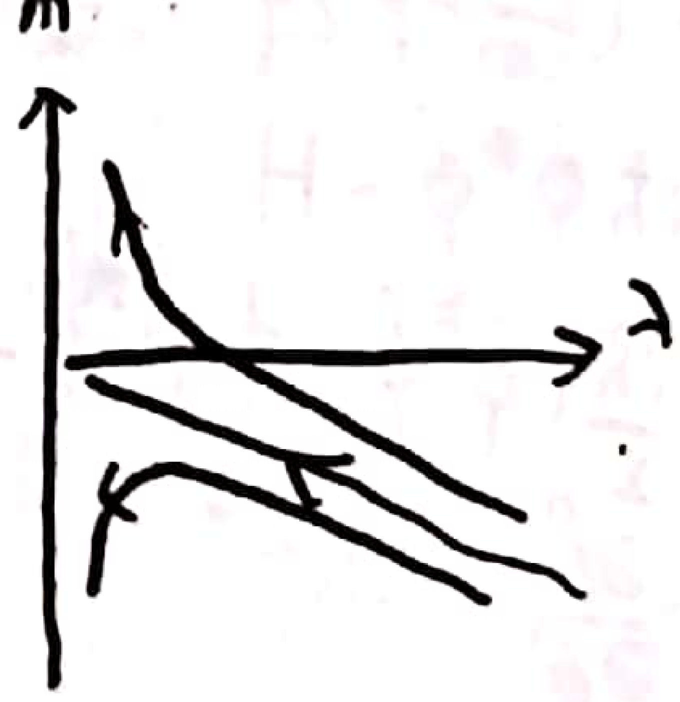
$$\frac{d}{dl}\begin{pmatrix}\delta\lambda \\ \delta m^2\end{pmatrix} = A^*\begin{pmatrix}\delta\lambda \\ \delta m^2\end{pmatrix} \Rightarrow \frac{d\vec{x}}{dt} = A\vec{x} \Rightarrow x = \sum_n x_n e^{\lambda_n t}.$$

由于系統若沒失穩定
性分析是線性分析.
分析結果僅適於
穩定點附近.

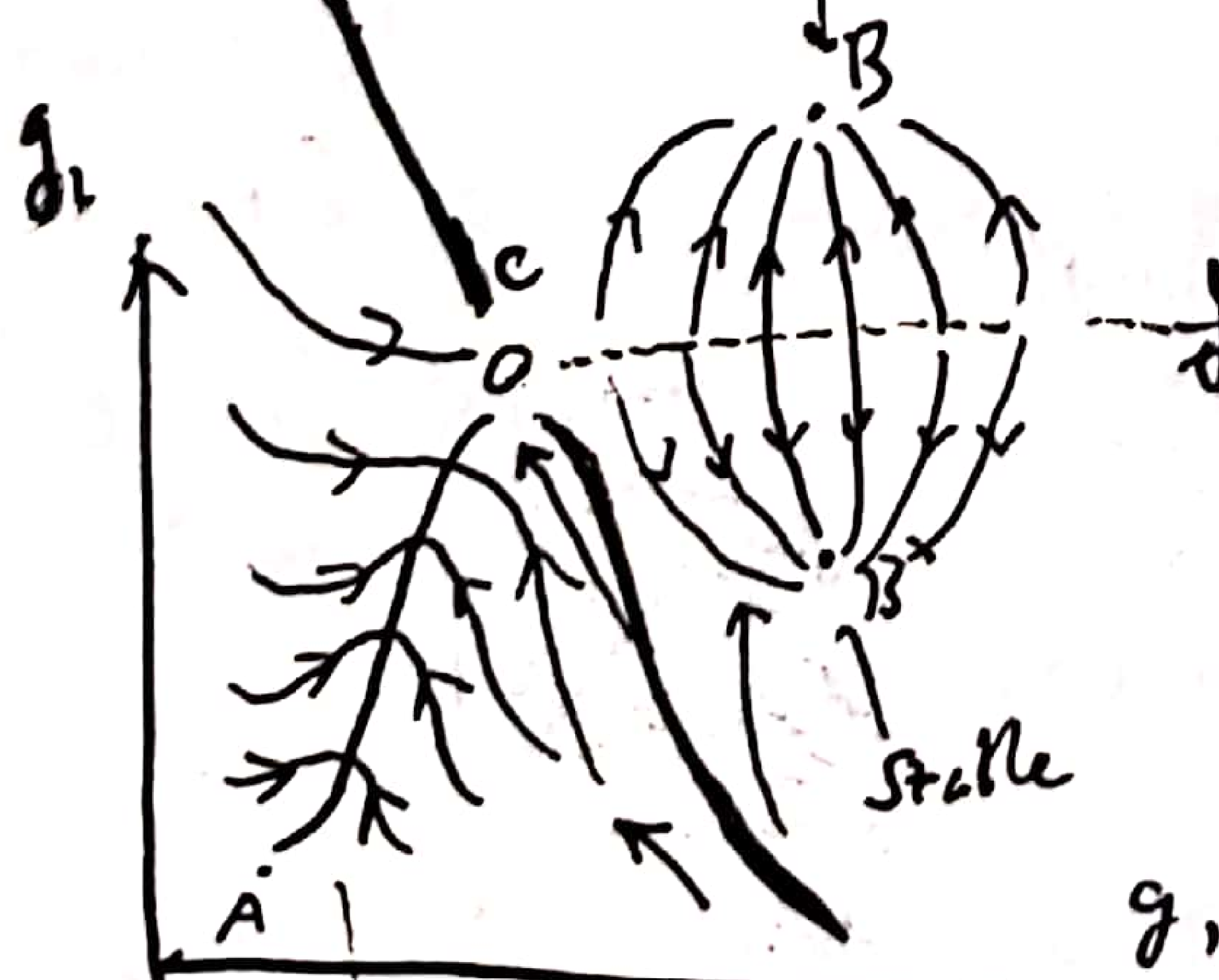


$V = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4$.





Stable.



相邊界.

① Fixed Point → g_i^* 決定 → V_i
Universality

② 邊界.

Stable Fixed Line (Low Energy Phys ~ 平均場).

BKT 相變

模型、

- 磁性（參 Nagosa）、
- BEC．

模型不同、物理一樣．

① 1d Ising Model 無相變
2d Ising Model 有相變

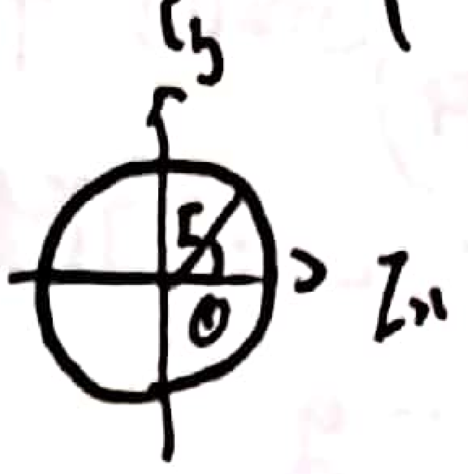
②. 1d XY Model XXZ Model 有相

③. 1d 原子、2d 磁性材料不穩定、
(Klein). (無長程序)

$\vec{I}_i = (I_i^x, I_i^y)$

$$H = \underbrace{-J\sum_{ij}\vec{I}_i\cdot\vec{I}_j}_{相互} - \sum_i h_i\cdot \vec{I}_i$$

$$= -JI^2\sum\cos(\theta_i-\theta_j) - h\sum_i\cos\theta_i.$$



$\theta_{i+1}-\theta_i = (\partial\theta)\cdot a.$

$$= -JI^2\cos(a\nabla\theta) - h\cos\theta.$$

$$= -JI^2 + \frac{J}{2}I^2(\nabla\theta)^2 - h\cos\theta.$$

相互作用 BEC 與場論（參文小剛 3.3）、

Schrödinger equation 的場論表述．

$$i\frac{\partial}{\partial t}\phi = (\underbrace{\frac{p^2}{2m}+\mu}_{\text{Boson Field}})\phi + \underbrace{\frac{g}{2}|\phi|^2\phi}_{\text{Interaction}}.$$

$$H = \phi^*(\frac{p^2}{2m}-\mu)\phi + \frac{g}{2}(\phi^*\phi)^2$$

$$\begin{cases}\mathcal{L} = i\hbar\phi^*\dot\phi - H\\ \mathcal{L}' = \frac{i\hbar}{2}(\phi^*\dot\phi - \phi\dot\phi^*) - H.\end{cases}$$

兩種 L 等價．$\mathcal{L}-\mathcal{L}' = \frac{i\hbar}{2}(\phi^*\dot\phi - \phi\dot\phi^*) = \frac{i\hbar}{2}\frac{\partial}{\partial t}(\phi^*\phi)$

$\mathcal{L}\to\mathcal{L}+\frac{d}{dt}f(q)$ 規範、

constraint quantization.

$$\begin{cases}\pi = \frac{\partial\mathcal{L}}{\partial\dot\phi} = i\hbar\phi^*\\ \pi^* = -i\hbar\phi \neq (\frac{\partial\mathcal{L}}{\partial\dot\phi^*}) = 0.\end{cases}$$

$Z = \int D\phi D\phi^* e^{iS}$, $S = \int dx dt\,\mathcal{L}$.

目的：獲得低能有效模型．$\phi\to\bar\phi+\delta\phi$.

$$\text{Dirac}\begin{cases}\phi = \sqrt{\rho+\delta\rho}\,e^{i\theta}\\ \phi = re^{i\theta}.\end{cases}$$

$\delta\rho$ 與 wave 有關．

$$\mathcal{L}=\frac{i}{2}(\phi^*\dot\phi-\phi\dot\phi^*)-\frac{1}{2m}(\partial\phi^*)(\partial\phi)+\underbrace{\mu|\phi|^2-\frac{V}{2}(|\phi|^2)^2}_{\text{只與}\rho\text{、}\delta\rho\text{有關，並打開gap.}}\sim\mu\rho-\frac{V_0}{2}(\rho^2+\delta\rho^2).$$

$$=\underbrace{-(\rho+\delta\rho)\partial_t\theta}_{\text{線性項}}.$$

$$-\frac{\rho}{2m}(\nabla\theta)^2-\frac{(\nabla\delta\rho)^2}{8m\rho}-\frac{V}{2}\delta\rho^2+\mathcal{L}_0(\rho)$$

$\phi=\sqrt{\rho+\delta\rho}\,e^{i\theta}$, $\mu|\phi|^2+\cancel{-\frac{V}{2}(|\phi|^2)^2}$

$\sim\int\mu(\rho+\delta\rho)$.

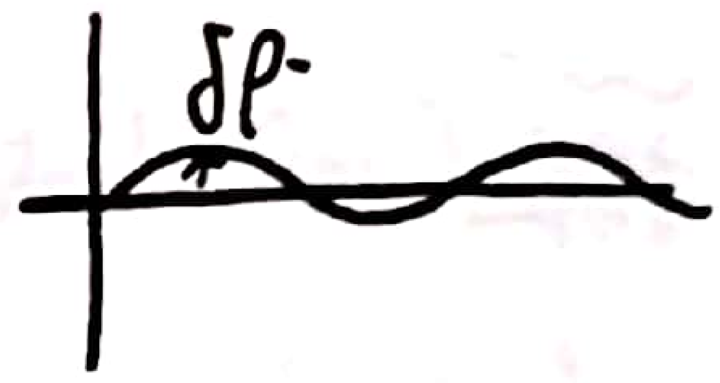
$\int\delta\rho\equiv0$.

$$\int(\rho+\delta\rho)^2=\rho^2+\delta\rho^2+\underbrace{2\rho\delta\rho}_{0}$$

忽略δρ，獲得不完全正確的 $\mathcal{L}_{effective}$.

$$\mathcal{L}_{effective}=-\rho\,\partial_t\theta-\frac{\rho}{2m}(\nabla\theta)^2$$
$$=-\frac{\rho}{2m}(\nabla\theta)^2 \quad (\Rightarrow)\text{石磁性}.$$

(何種相互作用會打開gap?
1. 吸引. Cooper Pair.
2. 密度波漲落 CDW.



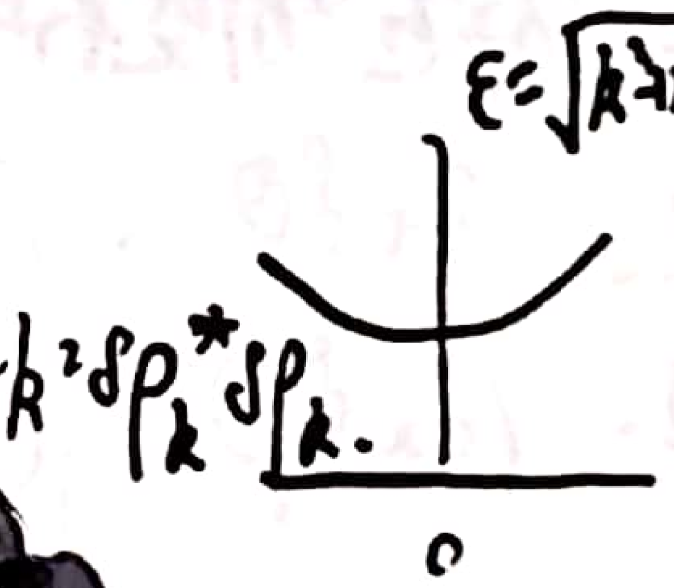
考慮δρ貢獻.

$(\partial_t\theta)^2-v^2(\partial_x\theta)^2$. 波動方程.

(方法. Gaussian Integral.
$\int D\phi\, e^{-A\phi^2+J\phi}$.)

$$\mathcal{L}=-\delta\rho\,\partial_t\theta-\frac{\rho}{2m}(\nabla\theta)^2-\frac{V}{2}\delta\rho^2-\underbrace{\frac{(\nabla\delta\rho)^2}{8m\rho}}_{\text{略去此項}}.$$

($k^2\delta\rho_k^*\delta\rho_k$



認為激發只發生在 $k\sim0$ 低能時)

或.

$$=-\frac{V}{2}\left(\delta\rho^2+\frac{2}{V}\partial_t\theta\,\delta\rho+\frac{4}{V^2}(\partial_t\theta)^2-\frac{4}{V^2}(\partial_t\theta)^2\right).$$

($\nabla\delta\rho\cdot\nabla\delta\rho$
$=\nabla(\delta\rho\nabla\delta\rho)-\delta\rho\nabla^2\delta\rho$.
捨棄.)

∴ 低能Model.

$$S_{eff}=\int dx\,dt\left(\frac{1}{2V}(\partial_t\theta)^2-\frac{\rho}{2m}(\nabla\theta)^2\right)$$

$\theta=\theta_k e^{i(\omega t-\vec{k}\cdot\vec{x})}$ (2D XY Model).

$$\frac{\omega^2}{2V}=\frac{\rho}{2m}k^2 \Leftrightarrow \omega^2=\frac{\rho V}{m}k^2 \Rightarrow \omega=\sqrt{\frac{\rho V}{m}}|k|.$$

以上結果強烈依賴於 $\phi=\sqrt{\rho+\delta\rho}\,e^{i\theta}$.

便不適用.

關聯

$\mathcal{L} = \frac{\chi}{2}\left((\partial_t\theta)^2 - v^2(\partial_x\theta)^2\right)$ (超流).

$\langle \phi^\dagger(x)\phi(0)\rangle = \rho\int D\theta\, e^{i\theta(x) - i\theta(0) - i\int dx\,dt\,\mathcal{L}}$

OR

$\mathcal{L} = (\nabla\theta)^2 + h\cos\theta$. (XY Model).

$\psi = \underbrace{\sqrt{\rho+\delta\rho}}_{密度}\ \underbrace{e^{i\theta}}_{相位}$ 普通的分解.

為何低維系統不穩定.

一維聲子鏈.

$\langle x_i^2\rangle = \frac{1}{L}\sum_k n_k \sim \cancel{\int^{\beta Z} dk \frac{1}{e^{\beta v k}-1}}$

↓ 磁性 $\int_0^{\beta Z} dk \frac{1}{e^{\beta v k^2}-1}$ → 紅外發散

$\langle M_i^2\rangle \sim \frac{1}{L}\sum_k n_k \sim \int \frac{1}{e^{\beta v k^2}-1} k\,dk$ → 發散

BEC. 低維分析(反證法).

又 3.3.5.

Real 光子 → $\frac{1}{k}$ → 電荷 V

Toy in 超流 聲子 → $\frac{1}{k^4}$ → ? Vortex.

若

$\mathcal{L} = \mathcal{L}(\delta\rho, \delta\theta)$. 存在哪些耦合項.

不存在的耦合項. $\partial_t\delta\theta$、$\partial_t\delta\rho$、$\cancel{\delta\rho\,\partial_t\theta}$ $\delta\rho\,\partial_t\delta\rho$. $\partial_x d\rho$. $(\delta\theta)^2$.

可能的耦合項. $(\partial_x\delta\rho)^2$ $(\partial_x\delta\theta)^2$ $\delta\rho\,\partial_t\delta\theta$ $\underbrace{(\delta\rho)^2}_{貢獻\ gap}$

$-x^2 + Ayx \Rightarrow -(x-\frac{Ay}{2})^2 + \frac{A^2y^2}{4}$

In. U(1) symmetry.

$\psi \to \psi e^{i\theta}$

$\psi^*\partial_t\psi \to (\partial_t\theta)^2$ 項.

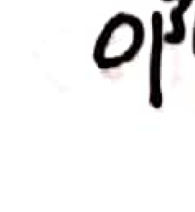
重整化群中.

最瀕發散度 相邊界 $D+A=0$

$dy = \underbrace{Dy\,dl}_{0階項} + \underbrace{Ay\,dl}_{修正項} = (D+A)y\,dl$.

若 $A = A(T)$ 則會影響相邊 從而產生相變.

$\frac{dy}{dl} = Dy \Leftrightarrow y \propto e^{Dl}$.

 $\sim \frac{\Lambda^d}{\Lambda^2+m^2}$  $\sim \frac{\Lambda^d}{(m^2+\Lambda^2)^2}$