These are the inhomogeneous Maxwell equations, with the current density $j^\nu = \bar{\psi}\gamma^\nu\psi$ given by the conserved Dirac vector current (3.73). As with ϕ^4 theory, the equations of motion can also be obtained as the Heisenberg equations of motion for the operators $\psi(x)$ and $A_\mu(x)$. This is easy to verify for $\psi(x)$; we have not yet discussed the quantization of the electromagnetic field.

In fact, we will not discuss canonical quantization of the electromagnetic field at all in this book. It is an awkward subject, essentially because of gauge invariance. Note that since $\dot{A}^0$ does not appear in the Lagrangian (4.3), the momentum conjugate to A^0 is identically zero. This contradicts the canonical commutation relation $[A^0(\mathbf{x}), \pi^0(\mathbf{y})] = i\delta(\mathbf{x}-\mathbf{y})$. One solution is to quantize in Coulomb gauge, where $\nabla \cdot \mathbf{A} = 0$ and A^0 is a constrained, rather than dynamical, variable; but then manifest Lorentz invariance is sacrificed. Alternatively, one can quantize the field in Lorentz gauge, $\partial_\mu A^\mu = 0$. It is then possible to modify the Lagrangian, adding an $\dot{A}^0$ term. One obtains the commutation relations $[A^\mu(\mathbf{x}), \dot{A}^\nu(\mathbf{y})] = -ig^{\mu\nu}\delta(\mathbf{x}-\mathbf{y})$, essentially the same as four Klein-Gordon fields. But the extra minus sign in $[A^0, \dot{A}^0]$ leads to another (surmountable) difficulty: states created by $a_{\mathbf{p}}^{0\dagger}$ have negative norm.*

The Feynman rules for calculating scattering amplitudes that involve photons are derived more easily in the functional integral formulation of field theory, to be discussed in Chapter 9. That method has the added advantage of generalizing readily to the case of non-Abelian gauge fields, as we will see in Part III. In the present chapter we will simply guess the Feynman rules for photons. This will actually be quite easy after we derive the rules for an analogous but simpler theory, *Yukawa theory*:

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Klein-Gordon}} - g\bar{\psi}\psi\phi. \tag{4.9}$$

This will be our third example. It is similar to QED, but with the photon replaced by a scalar particle ϕ. The interaction term contains a dimensionless coupling constant g, analogous to the electron charge e. Yukawa originally invented this theory to describe nucleons (ψ) and pions (ϕ). In modern particle theory, the Standard Model contains Yukawa interaction terms coupling the scalar Higgs field to quarks and leptons; most of the free parameters in the Standard Model are Yukawa coupling constants.

Having written down our three paradigm interactions, let us pause a moment to discuss what other interactions could be found in Nature. At first it might seem that the list would be infinite; even for a scalar theory we could write down interactions of the form ϕ^n for any n. But remarkably, one simple and reasonable axiom eliminates all but a few of the possible interactions. That axiom is that the theory be *renormalizable*, and it arises as follows. Higher-order terms in perturbation theory, as mentioned in Chapter 1, will involve

*Excellent treatments of both quantization procedures are readily available. For Coulomb gauge quantization, see Bjorken and Drell (1965), Chapter 14; for Lorentz gauge quantization, see Mandl and Shaw (1984), Chapter 5.

integrals over the 4-momenta of intermediate ("virtual") particles. These integrals are often formally divergent, and it is generally necessary to impose some form of cut-off procedure; the simplest is just to cut off the integral at some large but finite momentum Λ. At the end of the calculation one takes the limit $\Lambda \to \infty$, and hopes that physical quantities turn out to be independent of Λ. If this is indeed the case, the theory is said to be *renormalizable*. Suppose, however, that the theory includes interactions whose coupling constants have the dimensions of mass to some *negative* power. Then to obtain a dimensionless scattering amplitude, this coupling constant must be multiplied by some quantity of positive mass dimension, and it turns out that this quantity is none other than Λ. Such a term diverges as $\Lambda \to \infty$, so the theory is not renormalizable.

We will discuss these matters in detail in Chapter 10. For now we merely note that any theory containing a coupling constant with negative mass dimension is not renormalizable. A bit of dimensional analysis then allows us to throw out nearly all candidate interactions. Since the action $S = \int \mathcal{L}\, d^4x$ is dimensionless, $\mathcal{L}$ must have dimension $(\text{mass})^4$ (or simply dimension 4). From the kinetic terms of the various free Lagrangians, we note that the scalar and vector fields ϕ and A^μ have dimension 1, while the spinor field ψ has dimension 3/2. We can now tabulate all of the allowed renormalizable interactions.

For theories involving only scalars, the allowed interaction terms are

$$\mu\phi^3 \qquad \text{and} \qquad \lambda\phi^4.$$

The coupling constant μ has dimension 1, while λ is dimensionless. Terms of the form ϕ^n for $n > 4$ are not allowed, since their coupling constants would have dimension $4-n$. Of course, more interesting theories can be obtained by including several scalar fields, real or complex (see Problem 4.3).

Next we can add spinor fields. Spinor self-interactions are not allowed, since ψ^3 (besides violating Lorentz invariance) already has dimension 9/2. Thus the only allowable new interaction is the Yukawa term,

$$g\bar{\psi}\psi\phi,$$

although similar interactions can also be constructed out of Weyl and Majorana spinors.

When we add vector fields, many new interactions are possible. The most familiar is the vector-spinor interaction of QED,

$$e\bar{\psi}\gamma^\mu\psi A_\mu.$$

Again it is easy to construct similar terms out of Weyl and Majorana spinors. Less important is the *scalar QED* Lagrangian,

$$\mathcal{L} = |D_\mu\phi|^2 - m^2|\phi|^2, \qquad \text{which contains} \quad eA^\mu\phi\partial_\mu\phi^*,\ e^2|\phi|^2A^2.$$

This is our first example of a derivative interaction; quantization of this theory will be much easier with the functional integral formalism, so we postpone its