~~U = λ总~~

$$U = \lambda \int dx \, \frac{1}{6}\phi^3\hat\phi + \frac{1}{6}\hat\phi^3\phi + \frac{1}{4}\hat\phi^2\phi^2 + \frac{1}{24}\hat\phi^4 .$$

$$\langle e^{-u}\rangle_> = e^{-\underbrace{\langle u\rangle_c}_{\text{质量修正}} + \underbrace{\frac{1}{2}\langle u^2\rangle_c}_{\text{详见Kardar书、相互作用修正}}}$$

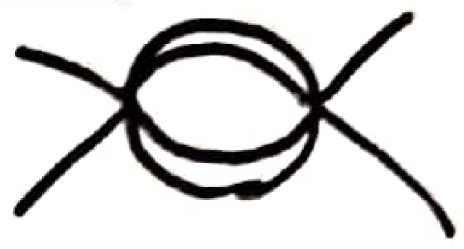
$$= -\delta m\,\phi^2 + \delta\lambda\,\phi^4 .$$

质量修正.

$\Delta m \propto \frac{\lambda}{V}\sum \frac{1}{k^2+m^2}$.

$\delta\Lambda \le |k| \le \Lambda$.

详见Kardar书、相互作用修正.



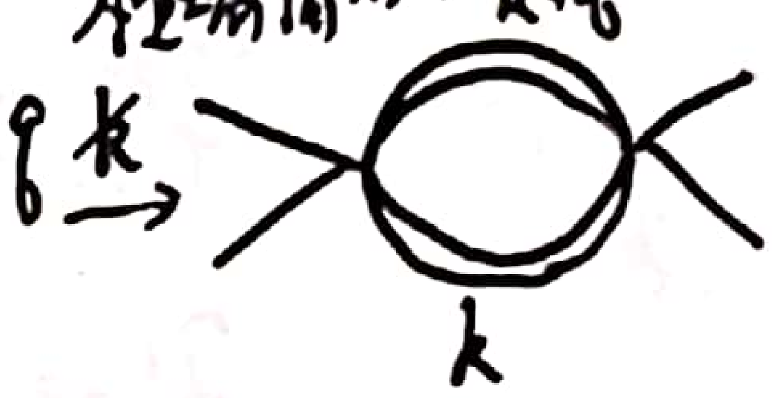
$$Z = \int D\phi \, e^{-S_{\text{effective}}[\phi]} .$$

做假设

$\delta\lambda \propto \frac{\lambda^2}{V}\sum_k \frac{1}{(k^2+m^2)^2}$

$$S_{\text{effective}}[\phi] = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 + \delta m\,\phi^2 - \frac{\delta\lambda}{4!}\phi^4 .$$

极端情形下



$|\delta\Lambda| \le k \le |\Lambda|$

$|\delta\Lambda| \le |k+q| \le |\Lambda|$.

修正 使质量部分增大.
相互作用 减小.

二阶微扰 使能量降低！相互作用减弱.

(超导 $\xrightarrow{\text{双/虚散射声子}}$ 库珀电对.).

(似飞去来)

(近藤效应 RKKY Interaction. Heisenberg ~~Model~~ Model . HTSC)

$\frac{e^2}{r} \to \frac{e^2}{r}e^{-\lambda r}$.

长程相互作用→短程相互作用.
减弱相互作用.

参 Yukawa Interaction.

推导.

$\frac{1}{2}\langle u^2\rangle_c$

$$= \frac{1}{2}\cdot\left(\frac{\lambda}{4}\right)^2 \int dx\,dx' \left\langle \hat\phi^2(x)\cdot\phi^2(x)\cdot\hat\phi^2(x')\,\phi^2(x)\right\rangle_c = \frac{\delta\lambda}{4!}\int dx\,\phi^4(x).$$

$$= \frac{1}{2}\left(\frac{\lambda}{4}\right)^2 \int dx\,dx'\,\phi^2(x)\,\phi^2(x')\left\langle \hat\phi^2(x)\,\hat\phi^2(x')\right\rangle_c$$

$= \cancel{\frac{1}{2}\cdot 2\left(\frac{\lambda}{4}\right)^2\int}$

($\hat\phi\hat\phi\hat\phi\hat\phi$

定义 $\langle\hat\phi(x)\,\hat\phi(x')\rangle = D(x-x')$.)

(KEY. How).

$\int dx\,dx' \to \int dx$

低能近似.

$= \frac{1}{2}\cdot 2\cdot(\frac{\lambda}{4})^2 \cdot \cancel{\frac{1}{V^4}} \int dx\, dx'\, \phi^2(x)\, \phi^2(x')\, D^>(x-x')\, D^>(x'-x)$　　定義 $\langle \hat\phi(x)\hat\phi(x')\rangle_c = D^>(x-x')$

$= \frac{1}{2}\cdot 2\cdot(\frac{\lambda}{4})^2 \cdot \frac{1}{V^4} \sum_k \sum_q \int dx\, dx'\, \phi_<(k_1)\phi_<(k_2)\phi_<(k_3)\phi_<(k_4)\, \underbrace{e^{-i(k_1+k_2)x - i(k_3+k_4)x'}}_{k_i \to 0} \times \underbrace{\left(\frac{1}{V^2}\frac{e^{iq_1(x-x')}}{q_1^2+m^2}\cdot\frac{e^{-iq_2(x'-x)}}{q_2^2+m^2}\right)_i}_{\text{高頻振盪}}$

$q_1+q_2=0$ 有貢獻.

$S_{effective} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(m^2+\delta m)\phi^2 + \frac{\lambda-\delta\lambda}{4!}\phi^4$.　　低能下,

$\int dx\, dx' \to V^2$.　$q_1+q_2=0$, $k\to 0$.

$\delta m = \frac{\lambda}{2}\cdot\frac{1}{V}\sum_k \frac{1}{k^2+m^2}$

$= \frac{\lambda}{2}\frac{1}{(2\pi)^3}\int_{b\Lambda\le k\le\Lambda} dk\, \frac{1}{k^2+m^2}$.

否則不能得到
$\int V(x-x')\phi^2(x)\phi^2(x')\, dx\, dx'$

k 補充條數

$q_1=-q_2$ 有效.

(Peskin. 維度正規化考量
Kardanoff/~~Shanker~~. 以上方法.
Shanker. $k_d = \frac{C_d}{(2\pi)^d}$
B. Simon)

$= \frac{\lambda}{2}\cdot\frac{1}{(2\pi)^d}\cdot\frac{1}{\Lambda^2+m^2}\left(V_d(\Lambda)-V_d(b\Lambda)\right)$

$= \frac{\lambda}{2}\cdot\frac{1}{(2\pi)^d}\frac{C_d\Lambda^d}{\Lambda^2+m^2}(1-b^d)$.　　$\begin{pmatrix} C_2=\pi \\ C_3=\frac{4}{3}\pi \\ C_4=\frac{\pi^2}{2} \end{pmatrix}$

$= \frac{\lambda}{2}k_d\frac{\Lambda^d}{\Lambda^2+m^2}(1-b^d)$.

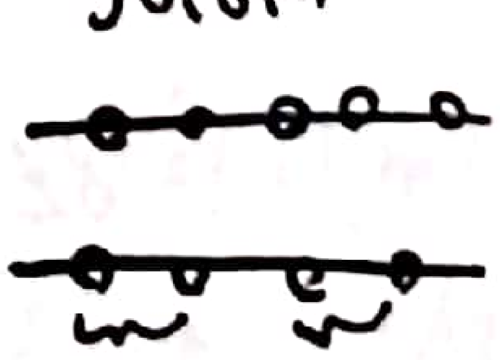
Peskin.

$\delta\lambda = -4!\cdot\frac{1}{2}\cdot 2\cdot(\frac{\lambda}{4})^2\frac{1}{V}\sum_k\frac{1}{(k^2+m^2)^2}$

$= -\frac{3}{2}\lambda^2 k_d\frac{\Lambda^d}{(\Lambda^2+m^2)^2}(1-b^d)$.　　二階微擾減小能量.

原始模型　$Z = \int[D\phi]_\Lambda e^{-S[\phi]}$.
$= \int[D\phi]_\Lambda e^{-S_{eff}[\phi]}$.
$= \int[D\phi]_{b\Lambda} e^{\cdots}$

Kardanoff
Block.



形式一致
空間不一.

需做 rescaling. 統一參數空間.

$\int dx\, \mathcal{L}_{effective} = S_{effective} = \int dx.\ \frac{1}{2}(1+\Delta B)(\partial_\mu\phi)^2$
$+\frac{1}{2}(m^2+\delta\phi)\phi^2$
$+\frac{1}{4}(\lambda+\delta\lambda)\phi^4$
$+\Delta C(\partial_\mu\phi)^4 + \Delta D\phi^6$.

Peskin P400　　New term. But trivial running coupling constant

$\int [D\phi]_{\Lambda}$. $|k| \le \Lambda$.

$\int [D\phi]_{b\Lambda}$. $|k| \le b\Lambda$. $\Rightarrow k = bk'$ $|k'| \le \Lambda$. 以保證 k 去 $k'x'$. 条量個.

$x = x'/b$. . $\phi(x) = \phi(x'/b) = A\phi(x')$.

$$\int [D\phi]_{b\Lambda}\, e^{-\int dx \left(\frac{1}{2}(1+\delta Z)(\partial_\mu\phi)^2 + \frac{1}{2}(m^2+\delta m)\phi^2 + \frac{1}{4!}(\lambda+\delta\lambda)\phi^4 + \Delta C(\partial_\mu\phi)^4 + \Delta D\phi^6\right)}$$

保證形式不變 fix point.

$$\int dx = b^{-d}\int dx'.$$

$$\int dx'\, b^{-d+2}\left(\partial_\mu'\phi\left(\tfrac{x'}{b}\right)\right)^2 \frac{1+\delta Z}{2}$$

$$= \underbrace{A^2 b^{-d+2}\left(1+\frac{\delta Z}{2}\right)}_{\frac{1}{2}} \int dx' \left(\partial_\mu'\phi(x')\right)^2 = \frac{1}{2}\int dx' \left(\partial_\mu'\phi(x')\right)^2.$$

$$\int dx\ \frac{m^2+\delta m}{2}\phi^2$$

$$= A^2 \left(\frac{m^2+\delta m}{2}\right)\cancel{b^{-d+2}}\, b^{-d}\int dx'\ \phi^2(x')$$

$$= \frac{1}{2}\underline{m^2(b)}\int dx'\ \phi^2(x') = \frac{1}{2}m'^2\int dx'\ \phi^2(x').$$

$$\int dx\ \frac{\lambda+\delta\lambda}{4!}\phi^4 = \frac{\lambda+\cancel{\delta\phi}\,\delta\lambda}{4!}A^4 b^{-d}\int dx\ \phi^4(x')$$

$$= \frac{1}{4!}\lambda^4(b)\int dx'\ \phi^4(x') = \frac{\lambda'}{4}\int dx'\ \phi^4(x').$$

⋮

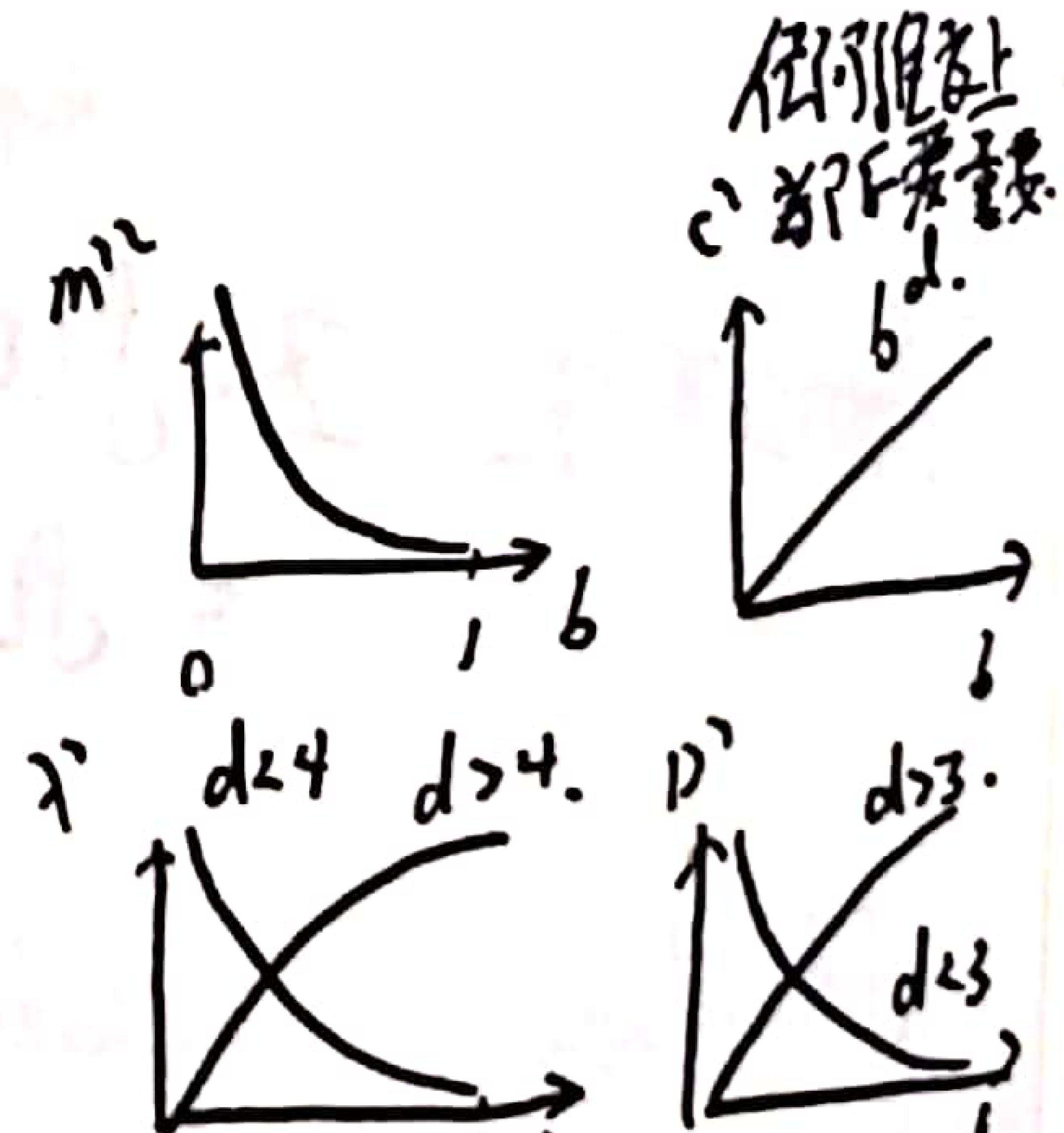
$$\Rightarrow m'^2 = (m^2+\delta m)(1+\delta Z)^{-1}b^{-2}$$

$$\lambda' = (\lambda+\delta\lambda)(1+\delta Z)^{-2}b^{d-4}$$

$$c' = (c+\delta c)(1+\delta Z)^{-2}b^{d}.$$

$$D' = (D+\delta D)(1+\delta Z)^{-3}b^{2d-6}$$

b 取值 $(0, 1)$
↑ 低能 ↑ 高能.



對於 $m'^2 = m^2(b)$.
取 $b = 1-dl = e^{-dl}$ } 連續化. $\delta Z = 1$. for ϕ^4 theory.

$$\Leftrightarrow m^2(b) = \left(m^2 + \frac{3}{2}\frac{k_d \Lambda^d (1-b^d)}{\Lambda^2+m^2}\right) b^{-2}.$$

$$\lambda(b) = \left(\lambda - \frac{3}{2}\lambda^2 k_d \frac{\Lambda^d}{(\Lambda^2+m^2)^2}(1-b^d)\right) b^{d-4}.$$

(取 $b = 1-dl$.
$1-b^d = 1-(1-dl)^d$
$= 1-(1-d\cdot dl)$
$= d\cdot(dl)$)

~~$\lambda(b) = (\lambda - \frac{3}{2}\lambda^2) k_d$~~

$$\lambda(b) = \left(\lambda - \frac{3}{2}\lambda^2 k_d \frac{\Lambda^d}{\Lambda^2+m^2}\cdot d\cdot dl\right)(1-(d-4)\cdot dl)$$

$$= \lambda - \lambda' dl.$$

($m^2(b) = m^2 - \frac{dm^2}{dl}\cdot(dl)$)

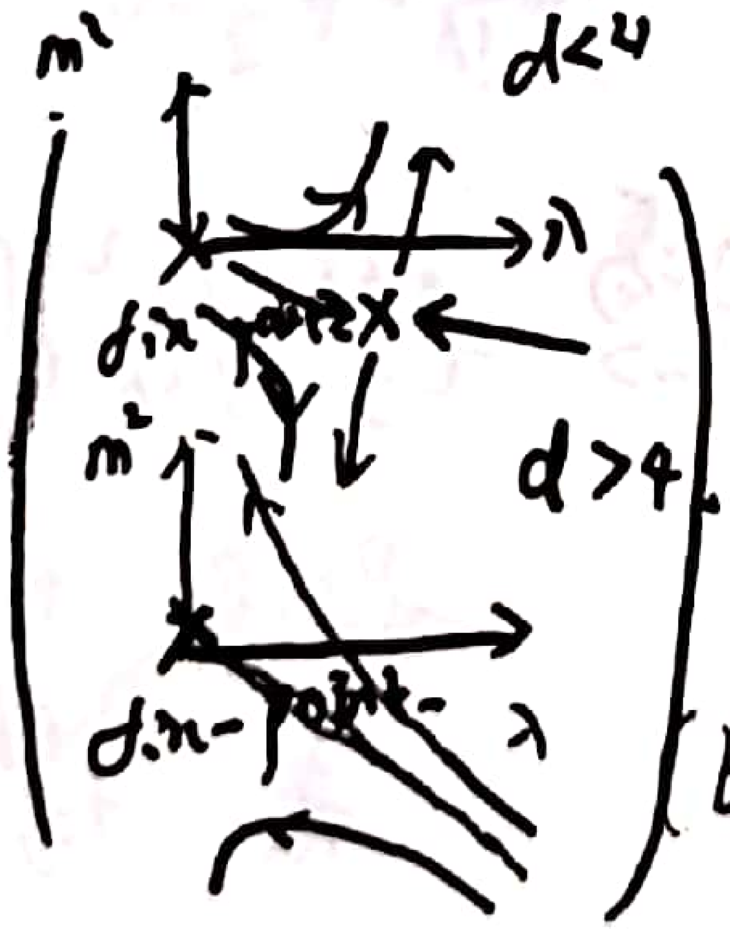
m^2 { 粒子物理. $m^2c^2 \to E$ gap.
凝聚態物理. μ. 化學勢. $\mu = m^2$.

$$\Leftrightarrow \lambda' = (d-4)\lambda + \cancel{\frac{3}{2}\lambda^2 k_d \frac{\Lambda^d}{(\Lambda^2+d)}}\ \frac{3}{2}\lambda^2 k_d \frac{\Lambda^d}{(\Lambda^2+m^2)^2} d.$$

(HW
$$\frac{d\lambda}{dl} = f_1(\lambda, m^2)$$
$$\frac{dm^2}{dl} = (f_2(\lambda, m^2)) - B\lambda^2$$
人為引入一項.)

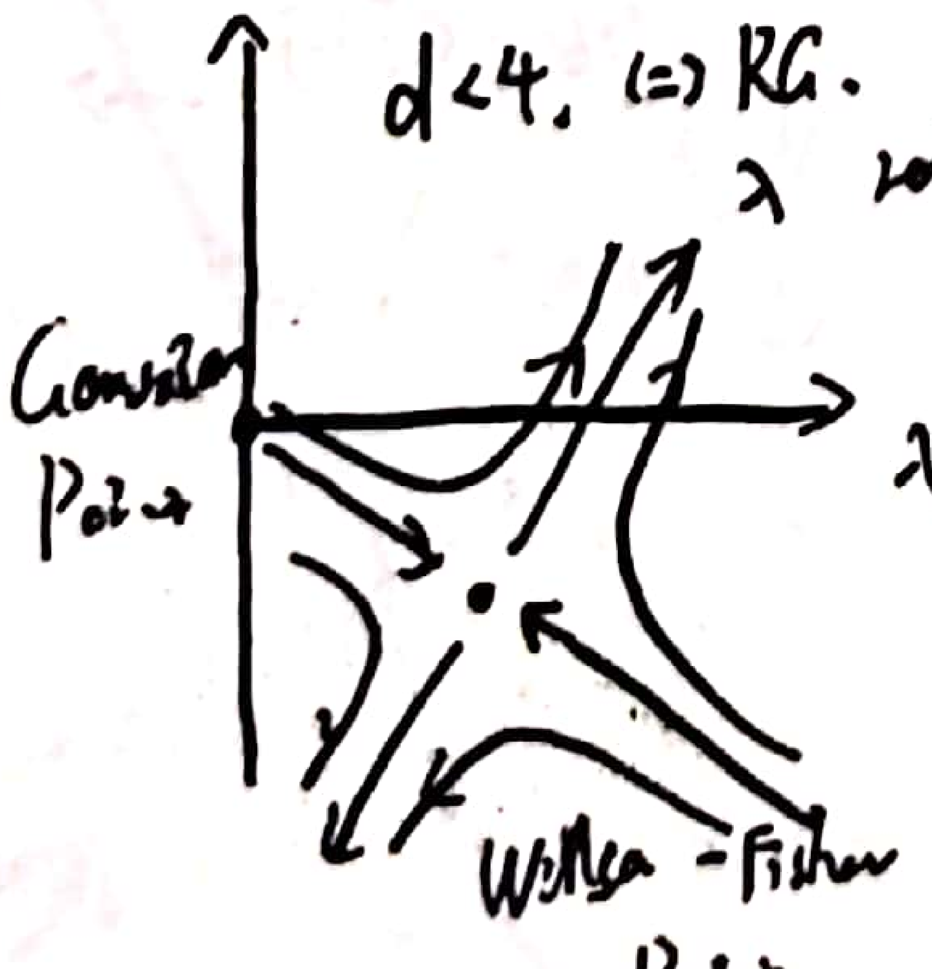
性質 ① $\lambda \to 0$.
$\lambda' = e^{(d-4)l}$. { $d<4$ 穩定. $d>4$ 發散.

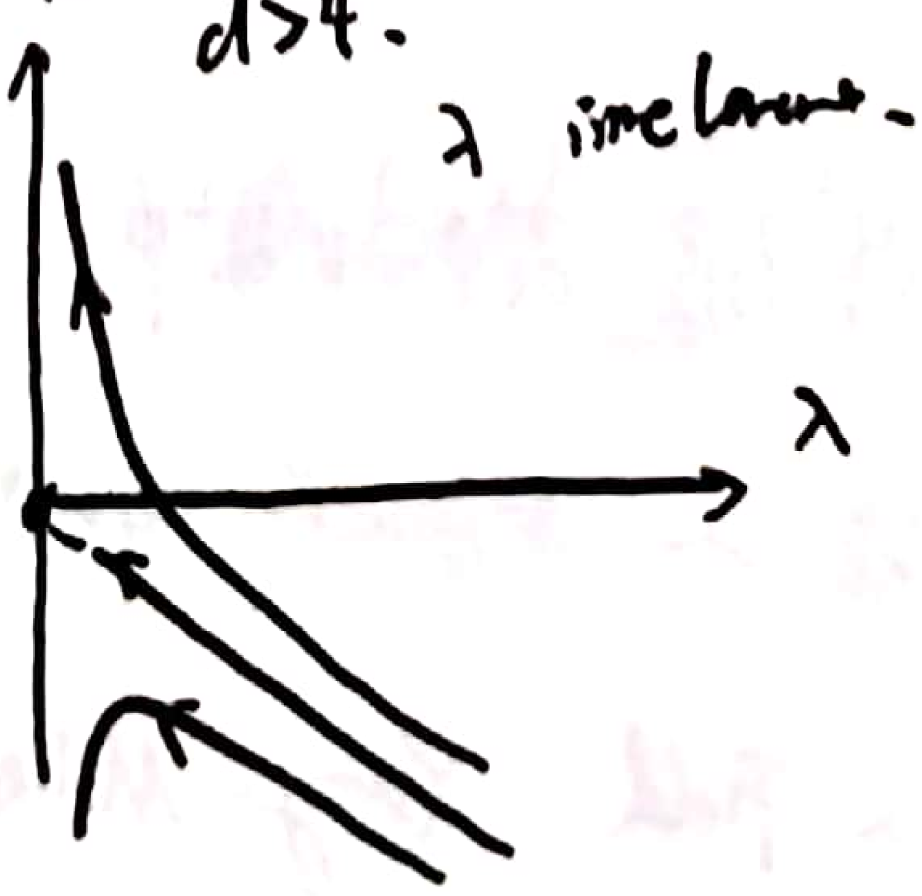
② $d<4$. 有2解.
$d>4$. 有1解.

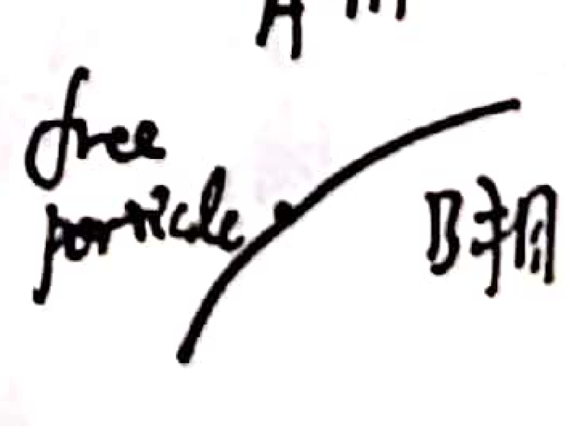


$\lambda^* = m^* = 0$. $\Leftrightarrow$ 高斯點. 自由粒子. $\frac{1}{2}(\partial_\mu \phi)^2$.
$\lambda^* \neq 0, m^* \neq 0$. $\Rightarrow$ Wilson-Fisher 點.

(B. Simon P455).







$$\begin{cases}[\phi]=1-\frac{d}{2}\\ [\lambda]=4-d\end{cases}$$

① $d>4$、$d<4$ 对 λ 有很大差别.
e.$d=2$、则 $[\phi]$ 无标度.

↓ BKT 相变. 允许 $\cos(\phi)$ 型势的出现.

$$\int D\phi\, e^{-\int \frac{1}{2}(\partial_\mu\phi)^2+g\cos(\phi)}$$

$$\begin{cases}\frac{d\lambda}{dl}=f_1(\lambda,m^2)=0\\ \frac{dm^2}{dl}=f_2(\lambda,m^2)=0.\end{cases}$$

λ^*, m^* 线性化.

$$\begin{cases}\lambda=\lambda^*+\delta\lambda\\ m^2=m^{*2}+\delta m^2.\end{cases}$$

$\cancel{\frac{d}{dl}(\delta\lambda}$ $\frac{d}{dl}\begin{pmatrix}\delta\lambda\\ \delta m^2\end{pmatrix}=A\begin{pmatrix}\delta\lambda\\ \delta m^2\end{pmatrix}$

李雅普诺夫稳定性分析

Spin Liquid

Field $\to$ $e^{i\int_i^j \vec{A}\cdot d\vec{l}}c_i^+c_j=e^{i\theta_{ij}}c_i^+c_j$.

可逆的过程.

$$\left.\begin{array}{l}H=J_{ij}\vec{S}_i\cdot\vec{S}_j\\ \vec{S}=f^+\sigma f\end{array}\right\}\xrightarrow[\text{平均场}]{f^+ff^+f} t_{ij}^{\sigma\sigma'}f_{i\sigma}^+f_{j\sigma'}+\Delta_{ij}^{\sigma\sigma'}f_{i\sigma}^+f_{j\sigma'}^+$$

Schwinger fermions.
or ~~...~~

最简单、t, Δ constant $\to$ magon.
若 $t_{ij}^{\sigma\sigma'}=te^{i\theta_{ij}^{\sigma\sigma'}}$
↓ $e^{i\theta_{ij}}c_i^+c_j$ $\to$ vector.
↓ hv.

场. 标量场/波 $\partial_t^2\phi=v^2\partial^2\phi$
↓
矢量场 ~ Maxwell equation.
↓
Matrix Field Yang-Milles Model
↓
Tensor ~ ?