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复习. 重整化 $\begin{cases} \text{抵消项}.\ \phi = \sqrt{Z}\phi_r,\quad m \to m_r \quad \lambda \to \lambda_r \quad \mathcal{L} = \mathcal{L}_r + \Delta\mathcal{L}. \\ \text{RG renormalisation Group (Wilson)} \end{cases}$

$\downarrow$

适合于讨论相变

Model: ① ϕ^4 Model:

$$Z = \int D\phi\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}$$

② 磁化

$$H(\vec{m}) = \frac{t}{2}(\nabla m)^2 + k\vec{m}^2 + \lambda \vec{m}^4$$

$$Z = \int D\vec{m}\, e^{-\beta\int H dx} = \int D\vec{m}\, e^{-\beta\int\left[\frac{t}{2}(\nabla m)^2 + k\vec{m}^2 + \lambda\vec{m}^4\right]dx}$$

③ BEC interaction.

$$H = \int dx\, \phi^\dagger\left(-\frac{\hbar^2}{2m}\nabla^2 - \mu\right)\phi + g|\phi|^4 \qquad \phi = \phi_1 + i\phi_2$$

$$Z = \int D\phi^\dagger D\phi\, e^{-\beta\int dx\, \phi^\dagger(-\nabla^2 - \mu)\phi + g|\phi|^4}$$

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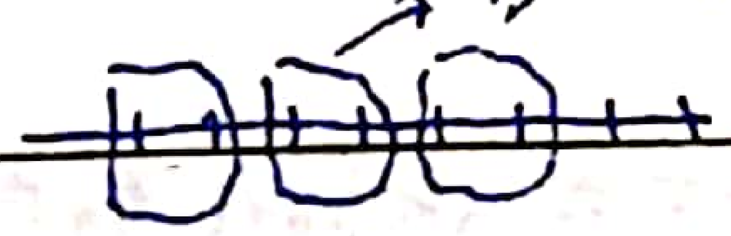
① [illegible] (Peskin P326)
② RG / Ising Model / [illegible] / [illegible] / P266
③ [illegible]

不变 scale invariant / 标度不变 $\phi(\lambda x) = \lambda^{-\nu}\phi(x)$

gapped 系统 $g(x) \sim e^{-x/\xi}$ ⇒ 没有关联 [illegible] ⇒ [illegible]

gapless 不变 $g(x) \sim \frac{1}{x^{\nu}}$ [illegible] ⇒ [illegible]

Block [illegible] 一个 [illegible]

Ising Model 

重整化群.

$R_{g'}R_g(\lambda) = R_{g'}(\lambda') = \lambda''$ $\quad R_{g'g}(\lambda) = \lambda''$ $\quad R_g R_{g'} = R_{gg'}$

Model ① ϕ^4 Model. ⇒ Peskin Chap 12 ⇒ $d=4$

$$Z = \int D\phi\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}$$

② [illegible]

$$H(\vec{m}) = \frac{t}{2}(\nabla m)^2 + k\vec{m}^2 + \lambda\vec{m}^4$$

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$$Z = \int D\phi^+ D\phi\, e^{-\beta\int dx\, \phi^+(-\nabla^2-\mu)\phi + g|\phi|^4}$$

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$$S=\int dx\,\mathcal{L}=\int d^dx\left[\frac{1}{2}(\partial_\mu\phi)^2-\frac{m^2}{2}\phi^2-\frac{\lambda}{4!}\phi^4+g_n\phi^n\right]$$

d为空间维度

$[x]=L$ $\qquad \left[L^d\left(\frac{\phi}{L}\right)^2\right]=0$ $\qquad dL-2L+2[\phi]=0$ $\qquad [\phi]=\frac{2L-dL}{2}$

$[\phi]=1-\frac{d}{2}$ $\qquad$ 令 $\Lambda L=1$. $\qquad d+2[m]+2[\phi]=d+2[m]+2-d=0$

$[m]=-1.$ $\qquad d+[g_n]+n\left(1-\frac{d}{2}\right)=0$ $\qquad [g_n]=n\left(\frac{d}{2}-1\right)-d$

$n=4$ 时 $[g_4]=[\lambda]=2d-4-d=d-4$ $\qquad \lambda\sim$ 或 $L^{d-4}\sim\Lambda^{4-d}$

$n=3$ 时 $\qquad [g_3]=\frac{d}{2}-3=\frac{1}{2}(d-6)$ $\qquad g_3\sim L^{\frac{1}{2}(d-6)}\sim\Lambda^{\frac{1}{2}(d-6)}$

记 $[g_n]\sim\Lambda^{-\nu}\sim L^{\nu}$ 则
$$\begin{cases}\nu<0 & \text{relevant} \Rightarrow \text{低能时不可忽略, 不能用 perturbation}\\ \nu=0 & \text{marginal}\\ \nu>0 & \text{irrelevant} \Rightarrow \text{低能时 } g_n\phi^n \text{ 不重要, 可用 perturbation Theory}\end{cases}$$

对于标度变换/有限重整化定义. $\qquad x\to x'=\lambda x$ $\qquad \frac{\partial}{\partial x'}=\frac{\partial}{\partial x}\frac{\partial x}{\partial x'}=\frac{1}{\lambda}\frac{\partial}{\partial x}$ $\qquad x=\frac{x'}{\lambda}$

$$S=\int dx\,(\partial_\mu\phi)^2=\int dx'\left(\frac{\partial\phi}{\partial x'}\right)^2=\lambda^d\frac{1}{\lambda^2}\int dx\left(\frac{\partial\phi}{\partial x}\right)^2=\lambda^{d-2}\int dx\left(\frac{\partial\phi}{\partial x}\right)^2$$

$$=\lambda^{d-2}\int dx\left(\frac{\partial\phi(\lambda x)}{\partial x}\right)^2$$ 定义 $\phi(\lambda x)=Z\phi(x)$

$\Downarrow$

$$=\lambda^{d-2}Z^2\int dx\left(\frac{\partial\phi(x)}{\partial x}\right)^2$$ $\qquad \lambda^{d-2}Z^2=1$ $\qquad Z=\lambda^{1-\frac{d}{2}}$

$[\phi]=[Z]=1-\frac{d}{2}$

$$\int g_4\phi^4dx=\int g_4'\phi^4(\lambda x)dx'=\lambda^dZ^4\int g_4'\phi^4(x)dx$$ $\qquad \lambda^dZ^4g_4'=g_4$

另 $g_4'=Z'g_4$ 则 $\qquad \lambda^dZ^4Z'=1$ $\qquad \lambda^dZ'\lambda^{4(1-\frac{d}{2})}=Z'\lambda^{4-d}=1$

$Z'=\lambda^{d-4}$. $[Z']=d-4$ $\qquad Z'\sim\Lambda^{4-d}$

表观发散度 (superficial divergence)

2种不同定义: $\begin{cases} g \sim L^{\nu} \\ g \sim \Lambda^{-\nu} < \begin{matrix} \Lambda\uparrow \ g\downarrow \ (\nu>0) \\ \Lambda\uparrow \ g\uparrow \ (\nu<0) \end{matrix} \end{cases}$

$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$

表观发散度 考虑耦合常数的量纲来决定 $g \sim L^{\nu}$ or $g \sim \Lambda^{-\nu}$

$S = \int d^dx\, \mathcal{L} = \int d^dx \left[\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + g_n \phi^n\right]$

$[x] = L$ $\quad \left[L^d\left(\frac{\phi}{L}\right)^2\right] = 0$ $\quad [\phi] = 1 - \frac{d}{2}$

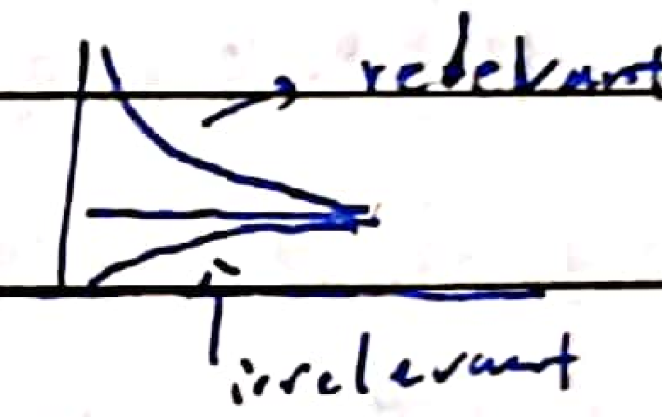
$2[m] + 2[\phi] + d = 0$ $\quad [\phi] = 1 - \frac{d}{2}$ $\quad \underline{[m] = -1}$ $\begin{cases} \text{能标越大 (小尺度 [illegible]) (尺度越小)} \\ \text{能标越小 (大尺度 [illegible]) (尺度越大)} \end{cases}$

$[g_n] + n[\phi] + d = 0$ $\quad [g_n] = n\left(\frac{d}{2} - 1\right) - d$

$n=4$ 时 $\Rightarrow 4\left(\frac{d}{2}-1\right) - d = d - 4$ $\quad [\lambda] = d-4.$ $\quad g_4 \sim L^{d-4} \sim \Lambda^{4-d}$

$n=3$ 时 $\quad 3\left(\frac{d}{2}-1\right) - d = \frac{d}{2} - 3 = \frac{1}{2}(d-6)$ $\quad g_3 \sim L^{\frac{1}{2}(d-6)} \sim \Lambda^{\frac{1}{2}(6-d)}$

量纲 L^{ν} $\Lambda^{-\nu}$ $\begin{cases} \text{relevant } \nu<0 \Rightarrow \text{低能时耦合强，不能用 perturbation} \\ \text{marginal } \nu=0 \\ \text{irrelevant } \nu>0 \begin{cases} \text{低能时不重要} \\ \text{perturbation works} \end{cases} \end{cases}$



标度变换/尺度变换. $x \to x' = \lambda x$

$S = \int dx\, (\partial_\mu\phi)^2 = \int dx' \frac{1}{2}\left(\frac{\partial\phi(x')}{\partial x'}\right)^2 = \lambda^{d-2}\int dx \left(\frac{\partial\phi(\lambda x)}{\partial x}\right)^2.$ #

定义 $\phi(\lambda x) = Z\phi(x)$

$\# = \lambda^{d-2} Z^2 \int dx \left(\frac{\partial\phi}{\partial x}\right)^2$ $\quad Z^2 = \lambda^{2-d}$ $\quad Z = \lambda^{1-\frac{d}{2}}$

$[\phi] = [Z] = 1 - \frac{d}{2}$ $\quad \int g_4 \phi^4 dx = \int g_4' \phi^4(x')\, dx'$

$= \lambda^d g_4' \int \phi^4(\lambda x) dx = \lambda^d g_4' Z^4 \int \phi^4(x) dx$

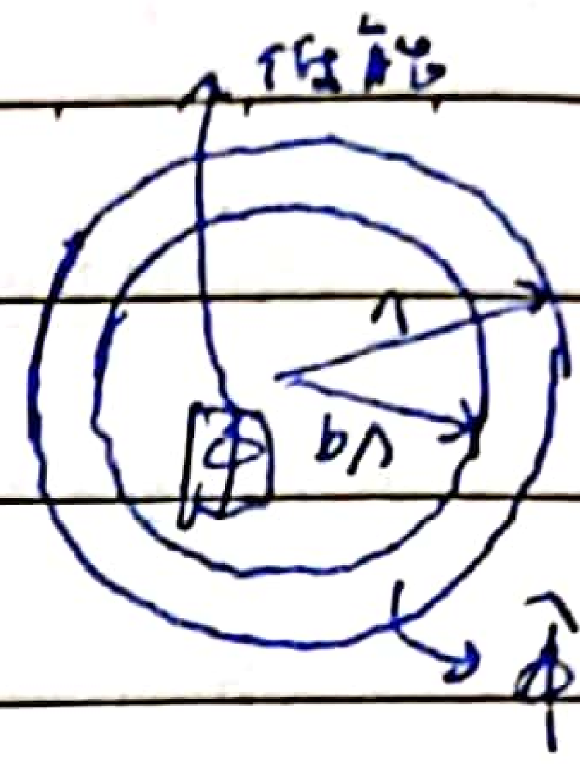
$\lambda^d Z^4 g_4' = g_4$ $\quad$ 令 $g_4' = Z' g_4$

$Z' \cdot Z^4 \lambda^d = 1$ $\quad Z' \cdot \lambda^{(1-\frac{d}{2})4 + d} = 1$ $\quad Z' = \lambda^{d-4} \sim \Lambda^{4-d}$

下面: $x' = \lambda x$ 时 尺度行为

注意，Z 是 partition function Z(之前) wick rotation 后的 $Z = \int D\phi\, e^{\int L dx}$

计算 $Z = \int D\phi\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}$



$$Z = \int [D\phi]_\Lambda\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}$$

$$[D\phi]_\Lambda = \cancel{\prod_{|k|<\Lambda} D\phi(k)}\ \prod_{|k|<\Lambda} D\phi(k) = \int [D\phi]_{b\Lambda}[D\hat\phi]_{b\Lambda\sim\Lambda}$$

$b = 1 - dl \approx e^{-dl}$ as $dl \to 0$

$$Z = \int [D\phi]_{b\Lambda}[D\hat\phi]_{b\Lambda\sim\Lambda}\, e^{-\int dx \frac{1}{2}(\partial_\mu(\phi+\hat\phi))^2 + \frac{m^2}{2}(\phi+\hat\phi)^2 + \frac{\lambda}{4!}(\phi+\hat\phi)^4}$$

第一项：$\int dx(\partial_\mu\phi + \partial_\mu\hat\phi)^2 = \int dx(\partial_\mu\phi)^2 + \int dx(\partial_\mu\hat\phi)^2 + 2\int dx\,\partial_\mu\phi\,\partial_\mu\hat\phi$

最后一项 经 Fourier Transformation：$\int (k_\mu e^{-ik\cdot x})(q_\mu e^{-iq\cdot x})$

由 δ 函数，只有 $k+q=0$ 才可能有贡献。 $|k|\le b\Lambda$，$b\Lambda \le |q| \le \Lambda$

因此交叉项的贡献为零。

$$\frac{1}{4!}(\phi+\hat\phi)^4 = \frac{1}{4!}(\phi^4 + \hat\phi^4 + 4\phi^3\hat\phi + 4\phi\hat\phi^3 + 6\phi^2\hat\phi^2)$$

$$= \frac{\lambda}{4!}\phi^4 + \frac{1}{4!}\hat\phi^4 + \frac{1}{6}\phi^3\hat\phi + \frac{1}{6}\phi\hat\phi^3 + \frac{1}{4}\phi^2\hat\phi^2$$

[注：在这里，我们对原来的 φ 加上了 高频部分 $\phi + \hat\phi$，并不是说……我们……，而是将原来 …… 中的 φ 分成低频的 φ 和高频部分 $\hat\phi$]

$$Z = \int [D\phi]_{b\Lambda}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{m^2}{2}\hat\phi^2 + \frac{\lambda}{4!}\hat\phi^4 + \frac{\lambda}{6}\phi^3\hat\phi + \frac{\lambda}{6}\phi\hat\phi^3 + \frac{\lambda}{4}\phi^2\hat\phi^2\right]}$$

由于 $\int f(x)p(x)dx = \frac{\int f(x)p(x)dx}{\int p(x)dx}\cdot\int p(x)dx = \left(\int p(x)dx\right)\langle f\rangle_{p 分布}$

$$Z = \int [D\phi]_{b\Lambda}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}$$

$$\cancel{\int [D\phi]_{b\Lambda\sim\Lambda} e^{-\int dx\,\frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{1}{2}}}\ \int [D\hat\phi]_{b\Lambda\sim\Lambda}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{m^2}{2}\hat\phi^2\right]}\ \left\langle -☆ \right\rangle$$

$$\int [D\hat\phi]_{b\Lambda\sim\Lambda}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{m^2}{2}\hat\phi^2\right]}$$

$$\langle ☆ \rangle = \int [D\hat\phi]_{b\Lambda\sim\Lambda}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{m^2}{2}\hat\phi^2 + \frac{\lambda}{4!}\hat\phi^4 + \frac{\lambda}{6}\phi^3\hat\phi + \frac{\lambda}{6}\phi\hat\phi^3 + \frac{\lambda}{4}\phi^2\hat\phi^2\right]}$$

$$Z = \int [D\phi]_{b\Lambda}\, e^{-S[\phi]}\, Z^0 \left\langle e^{-\int dx\left(\frac{\lambda}{4!}\hat\phi^4 + \frac{\lambda}{6}\phi\hat\phi^3 + \frac{\lambda}{6}\phi^3\hat\phi + \frac{\lambda}{4}\phi^2\hat\phi^2\right)} \right\rangle$$

$$Z^0 = \int [D\hat\phi]_{b\Lambda\sim\Lambda}\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{m^2}{2}\hat\phi^2\right]}$$

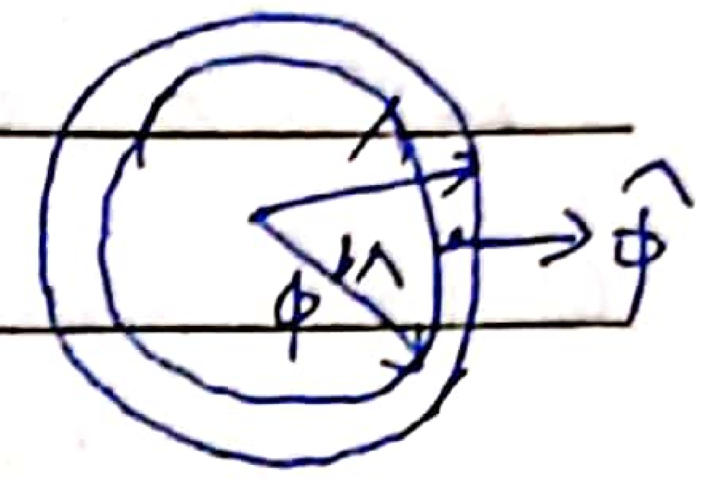
$$\langle e^{-u}\rangle = \langle 1 - u + \frac{1}{2!}u^2 \cdots\rangle = e^{-\langle u\rangle + \frac{1}{2}(\langle u^2\rangle - \langle u\rangle^2)}$$

$\parallel$

$$1 - \langle u\rangle + \frac{1}{2!}\langle u^2\rangle - \frac{1}{2!}\langle u\rangle^2 + \frac{1}{2!}\langle u\rangle^2 = e^{-\langle u\rangle + \frac{1}{2}(\langle u^2\rangle - \langle u\rangle^2)}$$

计算. $Z = \int D\phi\, e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]}$ ref: Peskin P393–P400

$Z = \int \underbrace{[D\phi]_\Lambda}_{\text{表示 } \prod_{|k|<\Lambda} D\phi(k)} e^{-\int dx\left[\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4\right]} = \int [D\phi]_{b\Lambda}\,[D\phi]_{\Lambda - b\Lambda}$



$b = 1 - dl \simeq e^{-dl}$ $\quad dl \Rightarrow 0^+$

$Z = \int [D\phi]_{b\Lambda} \int [D\phi]_{\Lambda - b\Lambda}\, e^{-\int dx \frac{1}{2}\underbrace{(\partial_\mu\phi + \hat\phi)^2}_{\text{无交叉项}} + \frac{m^2}{2}(\phi+\hat\phi)^2 + \frac{\lambda}{4!}(\phi+\hat\phi)^4}$

$(\partial_\mu\phi)^2 + (\partial_\mu\hat\phi)^2 + \int \partial_\mu\phi\,\partial_\mu\hat\phi\, dx = 0$

$\int k_\mu e^{-ikx} q^\mu e^{-iqx} dx$ $\quad |k| < b\Lambda \quad b\Lambda \le |q| \le \Lambda$. $k+q \neq 0$ 故交叉项为0

$= \int [D\phi]_{b\Lambda} e^{-\int S[\phi]} \int [D\phi]_{b\Lambda-\Lambda}\, e^{-\int dx \frac{1}{2}(\partial_\mu\hat\phi)^2 + \frac{m^2}{2}\hat\phi^2 + \frac{1}{4!}\hat\phi^4 + \frac{\lambda}{6}\phi^3\hat\phi + \frac{\lambda}{6}\phi\hat\phi^3 + \frac{\lambda}{4}\phi^2\hat\phi^2}$

$= \int [D\phi]_{b\Lambda} e^{-S[\phi]} Z^> \left\langle e^{-\int dx\left(\frac{\lambda}{4}\hat\phi^4 + \frac{\lambda}{6}\phi\hat\phi^3 + \frac{\lambda}{6}\hat\phi\phi^3 + \frac{\lambda}{4}\phi^2\hat\phi^2\right)}\right\rangle$

$Z^> = \int [D\hat\phi]\, e^{-\int dx (\partial_\mu\hat\phi)^2 + \frac{1}{2}m\hat\phi^2}$

$Z^> \langle e^{-u} \rangle$

利用 $\langle e^{-u}\rangle = e^{-\langle u\rangle + \frac{1}{2}(\langle u^2\rangle - \langle u\rangle^2)}$

$\underbrace{\phi\phi}_{|k|<b\Lambda}\underbrace{\hat\phi\hat\phi}_{|k|>b\Lambda}$

一阶近似 $\hat\phi \to$ const. $\langle \phi\hat\phi^3\rangle = 0$ $\quad \langle \phi^3\hat\phi\rangle = 0$

$\langle \phi^2\hat\phi^2\rangle$ $\quad e^{-\int dx\, \phi^2(x)\langle\hat\phi^2(x)\rangle}$

由关联 $\langle\phi(x)\phi(y)\rangle = g(x-y) \rightarrow$ const $g(0)$

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一阶贡献 $-\langle \int dx \frac{\lambda}{4!}\hat\varphi^4 + \frac{\lambda}{6}\hat\varphi^3\varphi + \frac{\lambda}{6}\varphi^3\hat\varphi + \frac{\lambda}{4}\varphi^2\hat\varphi^2 \rangle$

目的：得到低能有效的 $S[\varphi]$ 的修正（对 φ 无关的常数可以扔掉）

分析：① $\frac{\lambda}{4!}\hat\varphi^4$ 只有常数贡献（与 φ 无关） ② $\frac{\lambda}{4}\varphi\hat\varphi^3$ $\quad \varphi\hat\varphi\hat\varphi\hat\varphi$ $\begin{cases}\varphi\ (|k|<b\Lambda)\\ \hat\varphi\ (|k|>b\Lambda)\end{cases}$ 不为偏移

③ $\frac{\lambda}{6}\varphi^3\hat\varphi$ 同上 ④ 只有 $\frac{\lambda}{4}\varphi^2\hat\varphi^2$ 有贡献

$e^{-\langle \int dx \frac{\lambda}{4}\varphi^2\hat\varphi^2\rangle} = e^{-\int dx \frac{\lambda}{4}\varphi^2\langle\hat\varphi^2\rangle}$ $\qquad \langle\hat\varphi^2\rangle = \dfrac{\int [D\hat\varphi]_{b\Lambda\sim\Lambda}\, e^{-\int \frac{1}{2}(\partial\hat\varphi)^2+\frac{m^2}{2}\hat\varphi^2}\hat\varphi^2}{\int [D\hat\varphi]_{b\Lambda\sim\Lambda}\, e^{-\int\frac{1}{2}(\partial\hat\varphi)^2+\frac{m^2}{2}\hat\varphi^2}}$

$= g(x-y)$. $\qquad e^{-\int dx \frac{\lambda}{4}\varphi^2 g(0)\, g(x)} = \underset{\text{加入}}{e^{-\frac{\Delta m}{2}\int dx\, \varphi^2}}$ $\qquad \Delta m = \frac{\lambda}{2}\langle\hat\varphi^2\rangle$

Shell 的 主要结论：（只是在用大的有效作用能量来修正，低能区无贡献）

传播子：$\langle T\varphi(x)\varphi(y)\rangle = \frac{1}{V}\sum_k \frac{i e^{-ik(x-y)}}{k^2-m^2+i\varepsilon}$ $\qquad$ Wick 转动后为实的

$\frac{1}{V}\sum_k \longrightarrow \frac{1}{(2\pi)^d}\int dk$

$\Delta m = \frac{\lambda}{2}\langle\hat\varphi(x)^2\rangle_0 = \frac{\lambda}{2}\sum_{b\Lambda\le|k|\le\Lambda}\frac{1}{k^2+m^2} = \frac{\lambda}{2}\Big(\ \Big)$ shell 的积分 $\quad |k|<b\Lambda$

$= \frac{\lambda}{2}\frac{1}{(2\pi)^d}\int_{b\Lambda\le|k|\le\Lambda} dk\,\frac{1}{k^2+m^2} \xrightarrow[\text{(小量近似)}]{b\to 1\ k\ \text{变}} \frac{\lambda}{2}\frac{1}{(2\pi)^d}\frac{1}{\Lambda^2+m^2}C_d(\Lambda^d - b^d\Lambda^d)$

③ $V_d = C_d R^d$

将 $e^{-\frac{\Delta m}{2}\int dx\,\varphi^2}$ 代入原式得

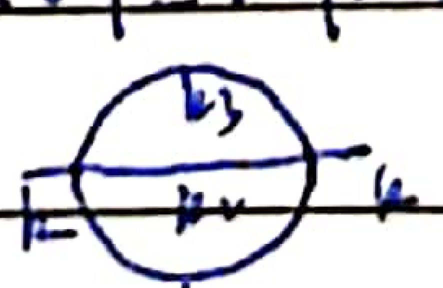
$$\begin{cases} Z = \int [D\varphi]_{b\Lambda}\, e^{-\int dx\, \frac{1}{2}(\partial\varphi)^2 + \frac{m^2+\Delta m}{2}\varphi^2 + \frac{\lambda}{4!}\varphi^4} \\ \quad |k|<b\Lambda \to k=bk' \to |k'|\le\Lambda \\ \int [D\varphi]_\Lambda\, e^{-\int dx\, \frac{1}{2}(\partial\varphi)^2 + \frac{m'^2}{2}\varphi^2 + \frac{\lambda'}{4!}\varphi^4} \end{cases}$$

$\Delta m = \frac{\lambda C_d}{2(2\pi)^d}\Lambda^d(1-b^d)$

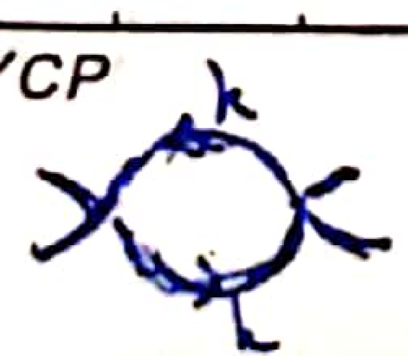
初步的 ~~重新~~ 标度 ... 的

同 $\begin{cases} H = -J\sum_i \sigma_{2i}\sigma_{2i+2} \quad 2i/2=i \quad 2i+2/2 = i+1 \\ \to -J\sum_i \sigma_i\sigma_{i+1} \\ \int [D\varphi] \end{cases}$ 重新标度 ... 2

一般 RG 计算中 大多只到二阶有效 ...

⊖ 计算的困难：  $\begin{cases} k_1+k_2+k_3=0 \\ 所以\ b\Lambda\le|k_i|\le\Lambda \end{cases}$ 这种积分太多，很难积

YCP

 计算时 $\frac{1}{V}\sum_k \lambda^2\frac{1}{(k^2+m^2)^2} = \frac{\lambda^2}{(2\pi)^d}\frac{C_d\Lambda^d}{(\Lambda^2+m^2)^2}(1-b^d)$

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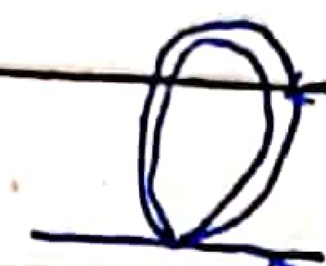
二阶项 $e^{-\frac{\lambda}{4}\int dx\, \phi^2(x) \langle \hat\phi^2(x)\rangle_0} = e^{-\frac{\delta m}{2}\int dx\, \phi^2(x)}$

$\delta m = \frac{\lambda}{2}\langle \hat\phi^2(x)\rangle_0$ shell 上这个修正对应diagram 一元更新

∴ 用到少数的更新式.

Textbook $\langle \hat\phi(x)\phi(y)\rangle = \underbrace{\frac{1}{V}\sum_k}_{\text{不对劲现}} \frac{ie^{-ik(x-y)}}{k^2-m^2+i\epsilon}$ p288 eq 8.27

$\delta m = \frac{\lambda}{2}\frac{1}{V}\sum_{b\Lambda<|k|<\Lambda} \frac{1}{k^2+m^2}$ $\quad \frac{1}{(2\pi)^3}\int dk$

$= \frac{\lambda}{2}$   shell

修正 $p m$ $\quad |k|<b\Lambda$.

求4 $\delta m = \frac{\lambda}{2}\frac{1}{(2\pi)^d}\int_{b\Lambda<|k|<\Lambda} dk \frac{1}{k^2+m^2} = \frac{\lambda}{2}\frac{1}{(2\pi)^d}\frac{1}{\Lambda^2+m^2}C_d(\Lambda^d - b^d\Lambda^d)$

总结. $Z = \int [D\phi]_{b\Lambda}\, e^{-\int dx \frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2+\delta m}{2}\phi^2 + \frac{\lambda}{4!}\phi^4}$

$|k|<b\Lambda$ $\quad k = bk' \quad k' \Rightarrow \frac{k}{b}$

$\cdot \int [D\phi]_\Lambda\, e^{-\int dx \frac{1}{2}(\partial_\mu\phi)^2 + \frac{k^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4}$

$H = -J\sum_i \sigma_{2i}\sigma_{2i+2} \xrightarrow{\div 2}$ $\quad 2i/2 = i$

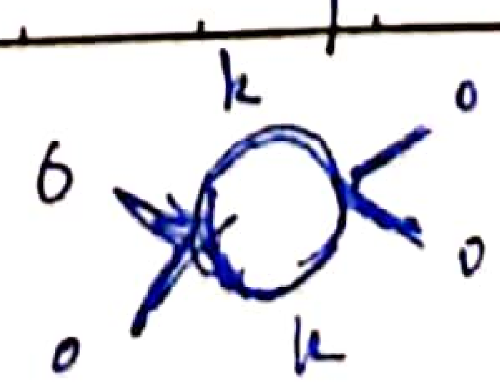
$\Rightarrow -J\sum_i \sigma_i\sigma_{i+1}$ $\quad 2i+2/2 = i+1$.

rescale $\begin{cases} |k| < b\Lambda \Rightarrow \left|\frac{k}{b}\right| < \Lambda \\ 2i \to 2i/2 = i \end{cases}$

二阶近似: $Z \approx \int d\alpha$ (Kardar)

	$\frac{1}{24}\hat\phi^4$	$\frac{1}{6}\hat\phi^3\phi$	$\frac{1}{4}\hat\phi^2\phi^2$	$\frac{1}{6}\hat\phi\phi^3$
$\frac{1}{24}\hat\phi^4$	$\hat\phi^8$ const	$\hat\phi^7\phi = 0$	$\hat\phi^6\phi^2$	$\hat\phi^5\phi^3 = 0$
$\frac{1}{6}\hat\phi^3\phi$	$\hat\phi^7\phi = 0$	$\hat\phi^6\phi^2$	$\hat\phi^5\phi^3 = 0$	$\hat\phi^4\phi^4\phi$
$\frac{1}{4}\hat\phi^2\phi^2$	$\hat\phi^6\phi^2$	$\hat\phi^5\phi^3 = 0$	$\hat\phi^4\phi^4$	$\hat\phi^3\phi^5 = 0$
$\frac{1}{6}\hat\phi\phi^3$	$\hat\phi^5\phi^3 = 0$	$\hat\phi^4\phi^6$	$\hat\phi^3\phi^5 = 0$	$\hat\phi^2\phi^6$ irrelevant

$6-d$



$\propto \sum_k \frac{\lambda^2}{(k^2+m^2)^2} = \frac{1}{V}\sum_k \frac{\lambda^2}{(k^2+m^2)^2}$