

主题: 重整化群

复习: 重整化
 重整化群方程 (RG)
 $\phi = \bar{\phi} \cdot \nu$, $m \rightarrow m \nu$, $\lambda \rightarrow \lambda \nu$, $h \rightarrow h \nu + \delta h$
 δh 有密切关系
 $RG(m, \lambda, h)$

适合于研究“相变” (Scale invariant)
 $q(\lambda, x) = \lambda^{-\nu} q(x)$

gapped $g(x) \sim e^{-x/\xi}$ (短程关联) $\Rightarrow$ 13 块
 gapless/相变点 $g(x) \sim \frac{1}{x^\nu}$ (长程相互作用) $\Rightarrow$ 细节无关

细节无关如 Ising model $\rightarrow$ Block 无关 (Kadanoff)
 重整化群方程 (RG) $\Rightarrow$ 去掉高能, 引入低能

不断乘入 $\Rightarrow$ 找 fixed point $\Rightarrow$ model 不变

课程开始: CMP 的书基本都是 RG

RG 方程: $R_g R_g(\lambda) = R_g(\lambda') = \lambda''$

$$\Rightarrow R_g g(\lambda) = \lambda'' \Rightarrow R_g R_g = R_g g \quad \text{但无逆}$$

Model d^4 model $\Rightarrow$ Peskin chap 12 $d=4 \Rightarrow$ LGM

Chap 12 model: $\vec{m}(x) \Rightarrow d=3$ (Rep: S-mom, textbook)
 (vector)

关于 LGM: well man, low: 标量场 $g_{ij}(\partial_\mu \Sigma^i)(\partial^\mu \Sigma^j) \rightarrow \text{Symm}$
 Peskin: $\phi = \phi_{cl} + \delta\phi = (b, \dots, 0) + \pi$

② BEC for 相互作用系统 $d=3$

目的: 解决 CMP 问题

比较: $d=3+1$
 $\int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \right] \quad (\phi^4)$

① $Z = \int D\phi e^{-\int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \right]}$
 $H(\vec{m}) = \frac{1}{2}(\nabla \vec{m})^2 + k \vec{m}^2 + \lambda \vec{m}^4$
 $Z = \int D\vec{m} e^{-\beta \int d^3x H(\vec{m})} = \int D\vec{m} e^{-\beta \int d^3x \left[\frac{1}{2}(\nabla \vec{m})^2 + k \vec{m}^2 + \lambda \vec{m}^4 \right]}$
 (BEC)

② $H = \int d^3x \left[\frac{1}{2}(\nabla \vec{m})^2 + k \vec{m}^2 + \lambda \vec{m}^4 \right]$
 $\phi \in \mathbb{C}$, $\phi = \phi_1 + i\phi_2$ (BEC interaction)
 $Z = \int D\phi^* D\phi e^{-\beta \int d^3x \left[\frac{1}{2}(\nabla \phi)^2 + k \phi^* \phi + \frac{\lambda}{4}(\phi^* \phi)^2 \right]}$

由 $h = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4$
 考虑双数态 (Peskin chap 10) 号数与能标之间的关系

2 种不同态: $g \sim L^\nu$ or $g \sim L^{-\nu}$

由 $S = \int d^4x h = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 \right]$

$[X] = [L] = 1 \dots [d\phi] = [L]^{-\frac{1}{2}} [dS] = 0 \therefore [h] = [L^{-d}] = -d$

又 $[\frac{1}{2}(\partial_\mu \phi)^2] = [\frac{1}{2}(\frac{d\phi}{L})^2] \Rightarrow [L]^{-\frac{1}{2}} [L]^{-\frac{1}{2}} = -d \Rightarrow [L]^{-1} = -\frac{d}{2}$

$[L]^{-\frac{1}{2}} [d^2] = -d = 2[L] + 2-d \Rightarrow [m] = -1$

$[L]^{-\frac{1}{2}} [d^4] = -d = [L] + 4-2d \Rightarrow [L] = d-4$

$[g_n \phi^n] = -d = [g_n] + n - \frac{n d}{2} \Rightarrow [g_n] = n(\frac{d}{2} - 1) - d$

(若 $n=3$, $[g_3] = 3(\frac{d}{2} - 1) - d = \frac{1}{2}(d-6)$)

$[m] = -1$ 意味着 $[m \sim L^{-1}]$ 与 d 无关 (维度)

其他 $g_n \sim L^{d-n} \sim L^{d-4}$
 $g_3 \sim L^{\frac{1}{2}(d-6)} \sim L^{\frac{1}{2}(6-d)}$

relevant $v \neq 0 \Rightarrow$ 此时很紧 $\Rightarrow$ 不能用近似

marginal $v = 0$

relevant $\checkmark \Rightarrow$ 重要

relevant $\checkmark \Rightarrow$ 重要

1 perturbation works

→ irrelevant

指数意义	指数意义	$\lambda \rightarrow \lambda' = \lambda X$	时的权值行为
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翻成英文: $\rightarrow$

$$S = \int dx \frac{1}{2} (\partial_\mu \phi)^2 = \int dx \frac{1}{2} \left(\partial \phi(x) \right)^2$$

$$X' = \lambda X, S \text{ 不变}$$

$$= \lambda d^{-2} \int dx \left(\frac{d\phi(x)}{dx} \right)^2$$

定义 $\phi(\lambda) = \lambda^2 \phi(\lambda)$ (保时重数)

$$= \lambda^{d-2} z^2 \left| dx \left| \frac{dy}{dx} \right| \right|^2$$

$$\Rightarrow Z^2 = \lambda^{2-d} \Rightarrow Z = \lambda^{1-\frac{d}{2}} \Rightarrow [Z] = [1 - \frac{d}{2}]$$

$$\int dx g_{\mu} \phi^{\mu} = \int g_{\mu}^i \phi^{\mu}(x) dx^i = \int \lambda^{\alpha} g_{\mu}^i \phi^{\mu}(x) dx^{\alpha}$$

$$g_4 = {}_4Z^4 \lambda dg_4' \Rightarrow \lambda dg_4' Z^4 = {}_4Z^4 \lambda dg_4'$$

$$\frac{1}{2} g'_4 = Z' g_4 \Rightarrow Z' Z^4 \lambda^d g_4 = g_4 \Rightarrow Z' = \lambda^{-d} \lambda^{2d-4}$$

$$\frac{1}{p-q} V \sim \frac{1}{p-q} \sqrt{d-4} \Rightarrow [\tilde{Z}] = d-4 \Rightarrow h-p = d-4 = \lambda d-4$$

2. 不計素因 (用已知 $1 \leq t \leq 2 \cdot 10^6$)

用板形门时,对发数反作预料

Ruitai Paper

Dose

part 7 / junction / with volatiles

$$- \int dx \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \right]$$

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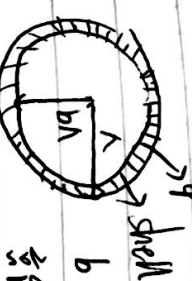
Fe^{2+} 氧化成 Fe^{3+} ⇒ $\text{Fe}(\text{OH})_3$ 沉淀 (Gibbs 自由能)

老行用の $\frac{1}{2}$ every

$$Z = \int [D\phi] e^{-\int dx \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4 \right]}$$

↓ II. $D(k)$ (A 是任意的, $\mathbb{Z}/3\mathbb{Z}$ 的任意 $\Rightarrow A$ 与 $\mathbb{Z}$ 有对应)

$\rightarrow$ 3.99% 与 ε 有关



不能与低能区合是人为主，不是自然

$$-dz \approx e^{-dz}, dz \rightarrow 0^+$$

$$\therefore \int [D\phi]_{\wedge} = \int [D\phi]_{\wedge \vee} [D\phi]_{\vee \wedge}$$

$$Z = \int \prod d\varphi_n \prod d\hat{\varphi}_{n+1} \rho - \lambda \sum_i \frac{1}{2} (d\varphi_i + d\hat{\varphi}_{i+1})^2 + \frac{m^2}{2} (\varphi + \hat{\varphi})^2 + \frac{\lambda}{a_i} (\varphi + \hat{\varphi})$$

$$\int_{\text{space}} (\hat{p}_m + \hat{p}_m^\dagger)^2 = \int_{\text{space}} \hat{p}_m^2 + (\hat{p}_m^\dagger)^2 + 2\hat{p}_m \cdot \hat{p}_m^\dagger dx = 0$$

$$\int \frac{d\mu}{\mu} \frac{d\mu}{d\mu} dx \quad \int k_n e^{ik_n y} e^{-ig_n x} dx$$

$$v = |z| = v_q \quad v_q > |k|$$

此方程有实根 $\Rightarrow k+q=0 \Rightarrow$ 所以无实根 $\rightarrow$ 所以

$$(\phi + \hat{\phi})^4 = \phi^4 + 4\phi^3\hat{\phi} + 6\phi^2\hat{\phi}^2 + 4\phi\hat{\phi}^3 + \hat{\phi}^4$$

Ruitai paper

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m}{2} \phi^2 + \frac{\lambda}{4!} \phi^4 \right]$$

$$= - \int \text{tr} \rho_{\text{br}} e^{-\text{St} \hat{\phi}} \int \text{tr} [\hat{\phi}]_{\text{shell}} e^{-\int d^4x \frac{1}{2} (\partial_\mu \hat{\phi})^2 + \frac{m^2}{2} \hat{\phi}^2 + \frac{g}{4!} \hat{\phi}^4 + \dots}$$

$$\Delta \psi^3 \hat{\psi} + \frac{\lambda}{6} \psi^3 \hat{\psi}^3 + \frac{\lambda}{6} \psi^2 \hat{\psi}^2 \rightarrow \text{4 2 2 和 } \psi^4 \text{ 有关 } (\lambda = \lambda_1)$$

$$\text{例 4 证明: } \int_{-\infty}^{+\infty} f(x) p(x) dx = \frac{\int_{-\infty}^{+\infty} f(x) p(x) dx}{\int_{-\infty}^{+\infty} p(x) dx} \cdot \int_{-\infty}^{+\infty} p(x) dx = c \cdot \int_{-\infty}^{+\infty} p(x) dx$$

Is this a perturbation? λ is a perturbation

$$\lambda + m \text{ is } (\text{peskm } | \text{mic } \lambda)$$

$$= |\text{IP}\rangle_{\text{bA}} e^{-\text{St}\phi} \sum_{\gamma} \langle e^{-|\alpha \times|} \hat{\phi}^{\alpha} + \frac{\alpha}{2} \hat{\phi}^{\beta} + \frac{\alpha}{2} \hat{\phi}^{\gamma} + \frac{1}{2} \hat{\phi}^{\delta} \rangle_{\text{bA}} \rightarrow \mathcal{V} \text{ for } \text{St}\phi \text{ and } \text{St}\phi \text{ and } \text{St}\phi$$

$$Z^1 = \int [D\phi] e^{-\int dx \frac{1}{2} (\partial \phi)^2 + \frac{m^2}{2} \phi^2}$$

$$= \int \Gamma(p)_{b_n} e^{-S(p)} z^n e^{-V} z^n$$

$$\text{利用 } \langle e^{-V} \rangle = e^{-\langle U \rangle + \frac{1}{2}(\langle U^2 \rangle - \langle U \rangle^2) + \dots}$$

$-(U^2) + \dots$
 ↓
 波动函数 δ^2

$$D_1 E_2 = 21 \times 10 = 210$$

-1π : $\hat{q}\hat{q}\hat{q}\hat{q} \equiv 0$ $\hat{q}\hat{q}\hat{q}\hat{q} \equiv 0$ $\hat{q}\hat{q}\hat{q}\hat{q}$ $\hat{q}^4$
 Cons. of $\hat{q}$

$$\dots e^{-\int dx \phi^2(x)} \langle \hat{\phi}^2(x) \rangle, \quad \hat{A} \langle \phi(x), \phi(y) \rangle = g(x-y) \quad \text{关联函数与距离有关}$$

$$\therefore \langle \hat{\phi}^2(X) \rangle \rightarrow \text{const.} \approx 9/10$$

$$\therefore \text{利用反例: } e^{-\frac{\lambda}{2} \int dx \dot{\phi}^2(x)} < \hat{q}(x) > = e^{-\frac{\Delta W}{2} \int dx \dot{\phi}^2(x)}$$

Russian paper $\Delta m = \frac{\Delta}{2} \langle \hat{\phi}(x) \rangle$, shell 重合性 $\Rightarrow$ 很多FD 的

王献之(王羲之)草书《东帖》

$\rightarrow \frac{1}{\sqrt{k}} \rightarrow$ 平均不会收敛

$\langle \Phi(x) \Phi(y) \rangle = \frac{1}{V} \frac{e^{-i\phi(x-y)}}{k^2 - m^2 + i\epsilon}$ (before Wick rotation)

$$\sum_{k=1}^{\infty} \|k\| = \frac{1}{p \wedge 3} \|H\|_{H^k} \Rightarrow \frac{1}{V} \sum_{k=1}^{\infty} \|k\| = \frac{1}{p \wedge 3} \sum_{k=1}^{\infty} \|k\|_{H^k} \quad \text{but } \frac{1}{V} \sum_{k=1}^{\infty} \|k\| \text{ finite}$$

$$\therefore \Delta m = \Delta \langle \hat{q}^2(k) \rangle = \frac{\Delta}{2} \sum_{k \in \text{shell}} \frac{1}{k^2 + m^2} = \frac{\lambda}{2} \bigcirc$$

(只是对shell求和)

$\frac{b}{a} \leq \frac{b}{a}$

$$= \frac{\Delta}{2} \frac{1}{(2\pi)^d} \int_{|b| \leq |b| \leq \Lambda} dk \frac{1}{k^2 + m^2} = \frac{\Delta}{2(2\pi)^d} \frac{1}{\Lambda^2 + m^2} C_d(\Lambda^d - b^d \Lambda^d)$$

$$V_d = C_d R^d \quad (\text{iter})$$

$$\chi_{\text{eff}}^2 = \frac{1}{2} \left(\chi^2 + \frac{m^2}{\Lambda^2} \phi^2 + \frac{\Lambda}{q_1} \phi \right)$$

resol $|k| \leq b \wedge \frac{1}{2} p = b p' \Rightarrow \left(\frac{p}{b}\right) \Rightarrow |k'| \leq n$

1) 若 $bX = X' \Rightarrow X \rightarrow bX$ $[\phi]$ 相左 & 右

注：部分主權不能改變 $\Rightarrow$ 不能自由地改變

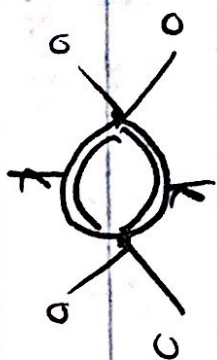
二、新亞細亞 (Zingher) Kander

Handwritten musical notation on a five-line staff. The notation includes various notes, rests, and accidentals, with some parts crossed out. Below the staff, there is a large 'X' and the text '1332' and '1332'.

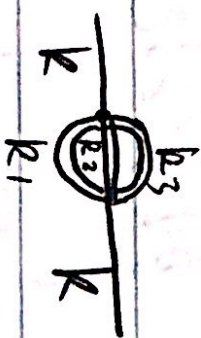
ϕ^2

$\hat{\psi}^2 \psi^6$ 可产生新相互作用 $\lambda_6 \psi^6$

used in



$$\alpha^{-1} \frac{1}{\sqrt{k}} \lambda^2 \frac{1}{(k^2+m^2)^2} = \frac{\Lambda^2}{\epsilon n, d} \frac{1}{(1+\mu^2)^2} \frac{1}{(1+\mu^2)^2} \frac{1}{(1+\mu^2)^2}$$



$$b \in [k_1] \leq \Lambda$$

$$k_1 + k_2 + k_3 = 0$$



自由能 k 有关一定含对 k 的修正

$$\text{ref. } \int [d\phi] = 1 - \frac{d}{2}$$

$$[\Lambda] = k \cdot d$$

$\frac{d}{2} d = 4 \Rightarrow \phi^4$ marginal

$d = 2 \Rightarrow [d\phi] = 0 \Leftrightarrow \text{sm}[\phi]$ 这是允许的

$\Rightarrow$ BKT 相变 (Thouless)