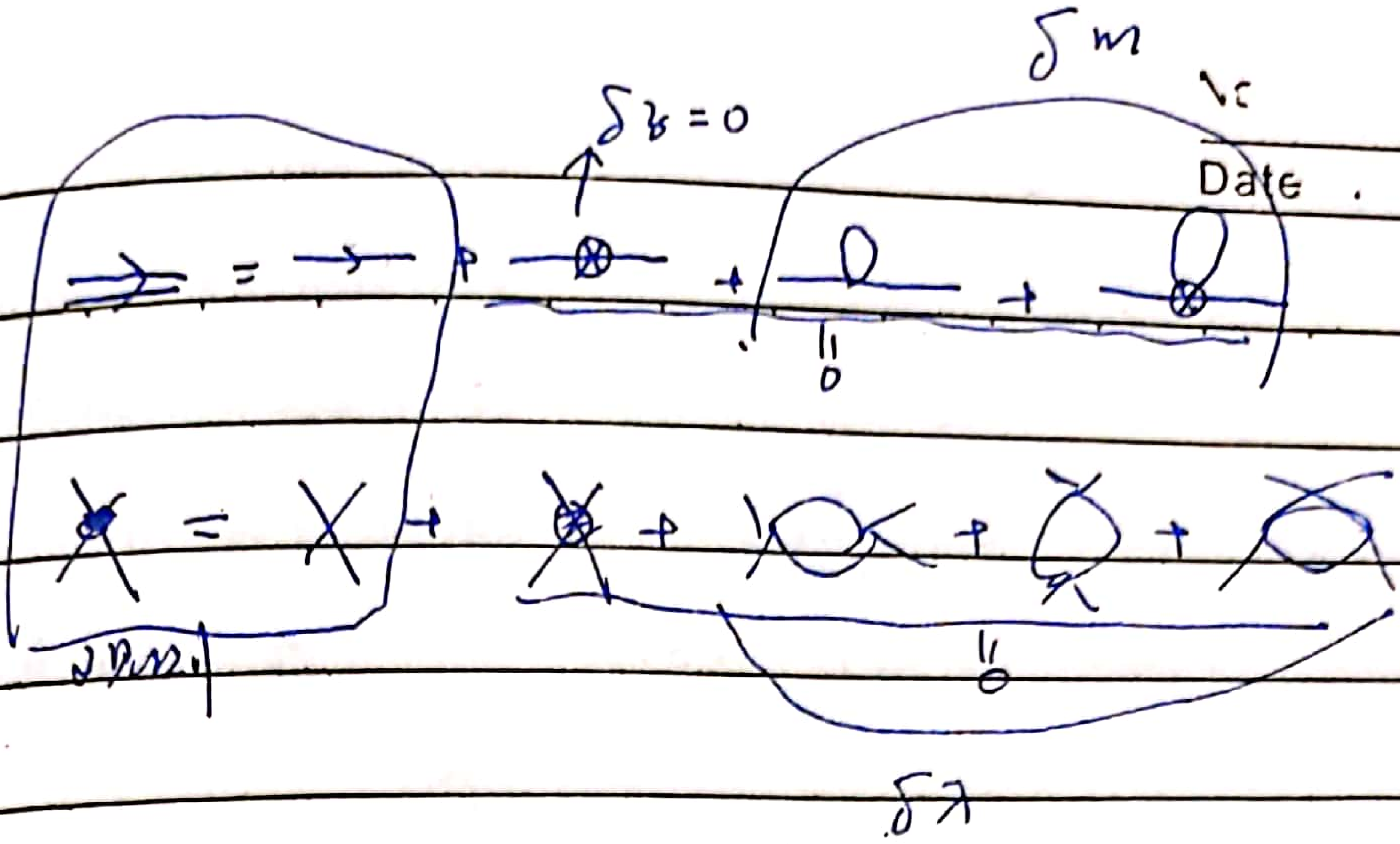
Date



2D δ(r) Potential · bound state $-E$

$\lambda \sum_k \frac{1}{k^2+E} = 1 \qquad \frac{1}{\lambda_\Lambda} = \sum_{|k|\le\Lambda} \frac{1}{k^2+E}$

bare perturbation theory

renormalised perturbation theory

$\frac{1}{\lambda} - \frac{1}{\lambda_R} = \sum_k \left(\frac{1}{k^2+E} - \frac{1}{k^2+\mu}\right)$

$\frac{1}{\lambda_R} = \sum_k \frac{1}{k^2+\mu}$

ϕ^4 theory $\quad \mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4}_{\text{bare lagrange}} \quad \Lambda,\ m_\Lambda\ \lambda_\Lambda$

perturbation

$\frac{1}{k^2-m_R^2} = \frac{1}{k^2-m^2} + \frac{1}{k^2-m^2}\sum \ldots \frac{1}{k^2-m^2} + \cdots$

$m_R^2 = m^2 + \sum(k, m^2)$

λ_R

$\lambda_R = \lambda + \ldots \quad \propto \sum \frac{1}{k^2+m^2}\frac{1}{k^2+m^2}$

$\propto \ln \Lambda$

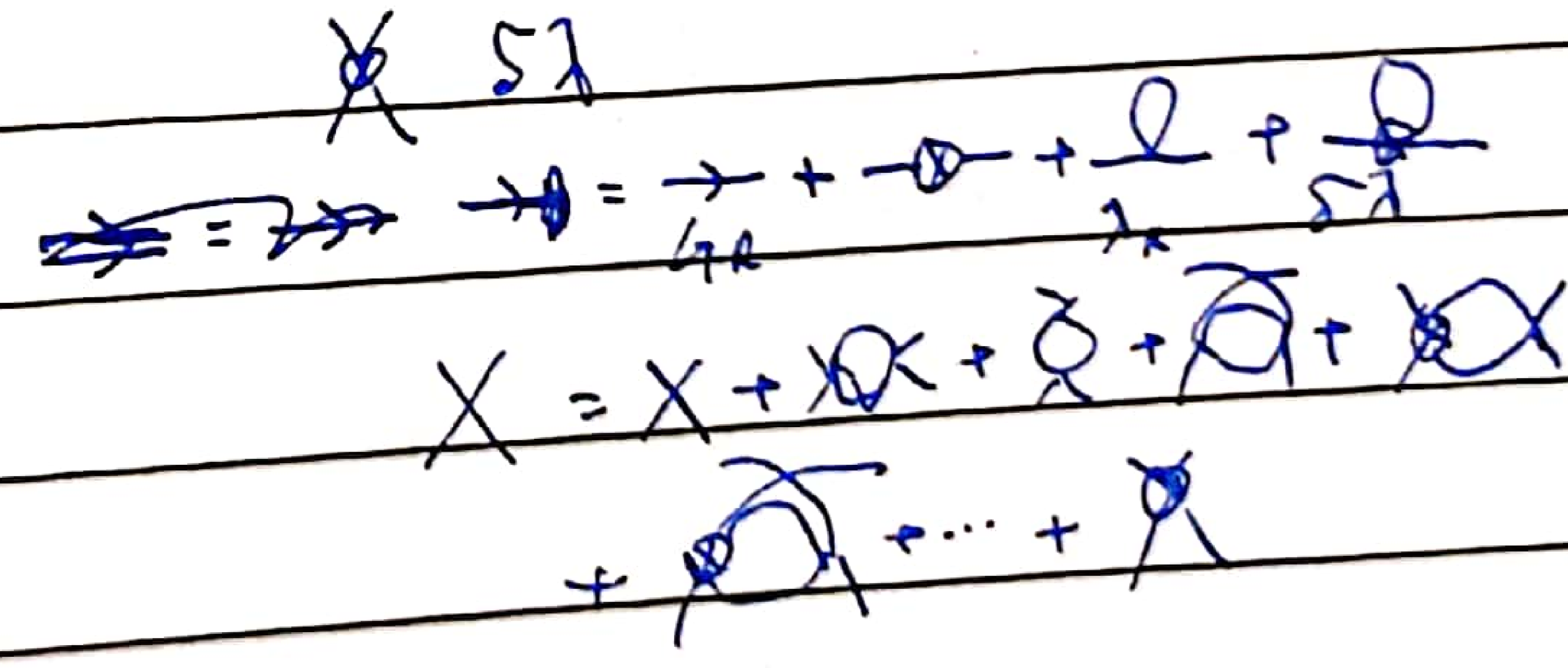
$\phi = \sqrt{Z}\,\phi_R$

$\propto \frac{Z}{k^2+m_R^2}$

$\mathcal{L} = \mathcal{L}_R + \underbrace{\delta\mathcal{L}}_{\text{counterterm}}$

$\delta\mathcal{L}_2 = (\delta_Z p^2 - \delta m)$

$\delta\lambda$



Casimir 力

Zeta 函数 $1+2+3+\cdots+n = -\frac{1}{12}$

E_g

$p_n = h/\lambda_n \qquad \lambda_n = \frac{a}{n} \qquad p_n = \frac{h}{a}n$

$E = pc = \frac{hc}{a}\cdot n$

$E_g = \sum \frac{\hbar c}{2}\sum_{n=1}^{\infty}\left(\frac{n\pi}{a}\right)\left[e^{-n/\Lambda}\right]$

$= \infty - \frac{\pi\hbar c}{24a}$

$-\frac{\partial E_g}{\partial x} = F = \frac{\pi\hbar c}{24a^2}$

No.
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重整化群. (群=群\逆)

Renormalization Group (R.G)

K. Wilson (Gellmann's student) 　　　尺度变换

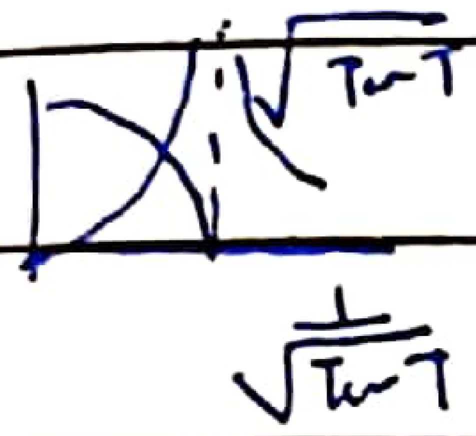
Kondo Effect. / lattice gauge theory

目的: 处理多尺度问题的方法. 　　　effective potential / effective action
T.

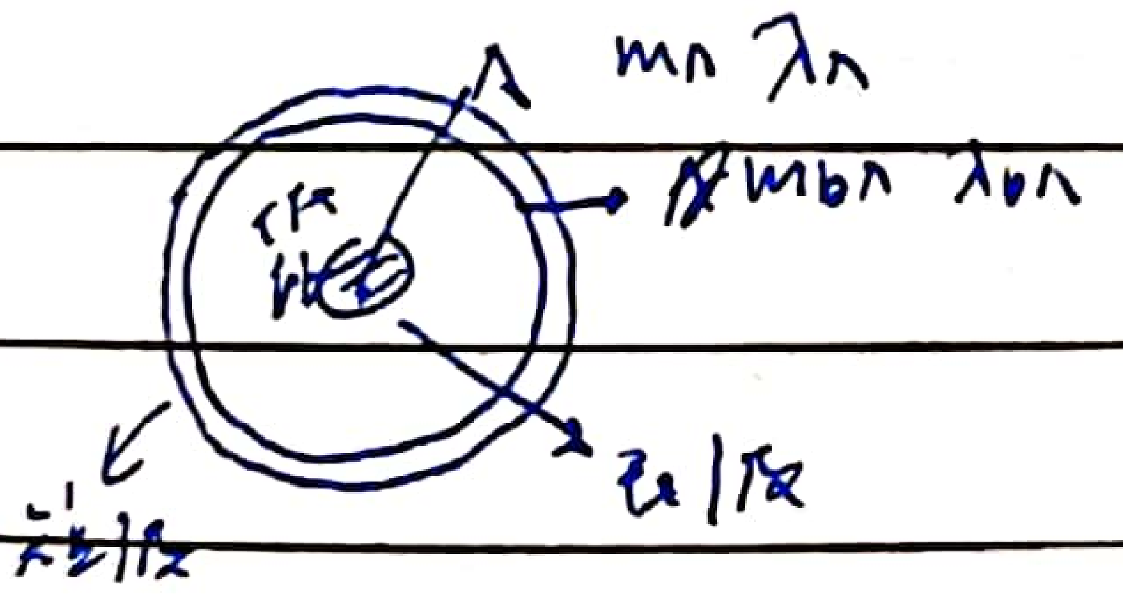
Callan-Symanzik Eq

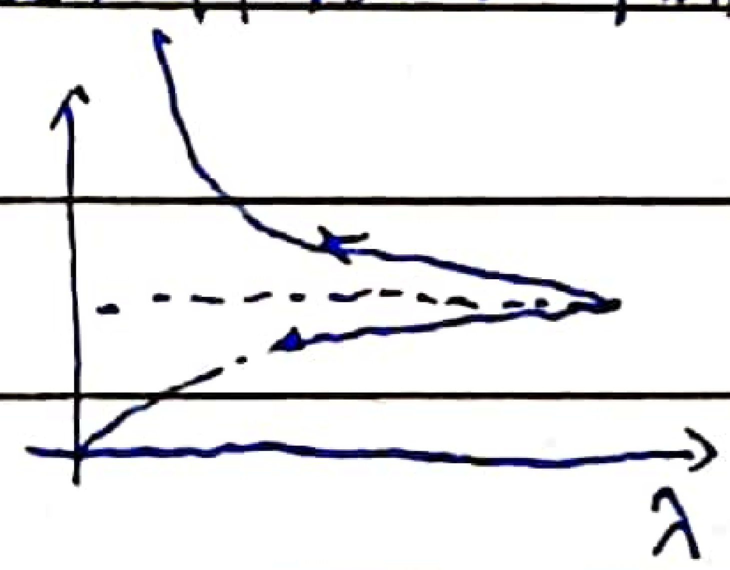
$f(\lambda x) = \lambda^{\nu} f(x)$ 　　$\langle s(x) s(0)\rangle \sim \frac{1}{x^{a}}$ 　gapless

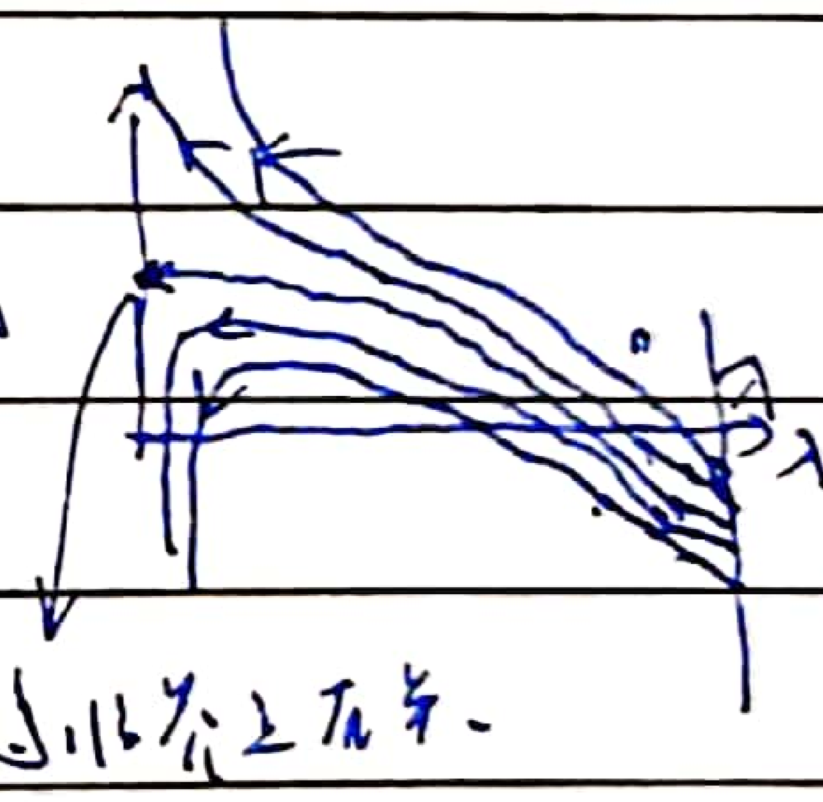
$\langle S(\lambda x) S(\lambda \cdot 0)\rangle \sim \lambda^{-a} \frac{1}{x^{a}}$

Landau.  $-a\Delta^2 + b\Delta^4$ 　Landau 标度理论失效.

改变观测尺度 Λ, 观察体系随尺度的变化



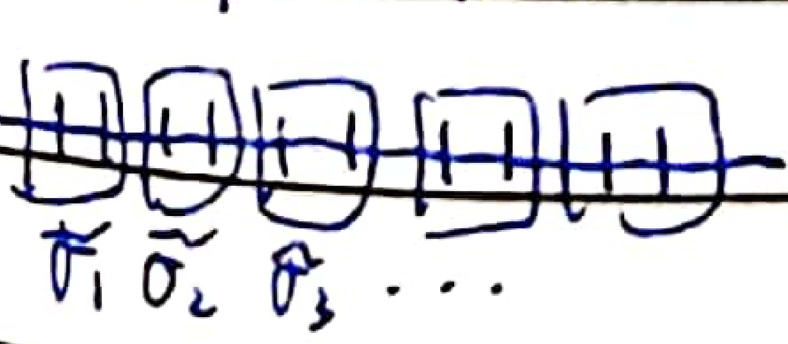




Ising Model.

$H = -J\sum_i \sigma_i \sigma_{i+1}$ 　　$\sigma_i = \pm 1$

$Z = \mathrm{Tr}\, e^{-\beta H} = \mathrm{Tr}\, e^{\beta J(\sum_i \sigma_i \sigma_{i+1})}$

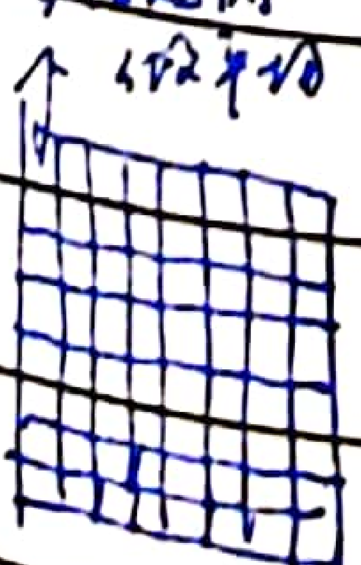


$= \mathrm{Tr}_{\text{奇}} \left(\mathrm{Tr}_{\text{偶}}\, e^{\beta J \sum_i \sigma_i \sigma_{i+1}} \right)$

$= \mathrm{Tr}_{\text{奇}}\, e^{\beta J' \sum_i \sigma_{2i+1}\sigma_{2i+3}}$

$\sum_{\sigma_2 = \pm 1} e^{\beta J(\sigma_1\sigma_2 + \sigma_2\sigma_3)} = e^{\beta J(\sigma_1+\sigma_3)} + e^{-\beta J(\sigma_1+\sigma_3)}$

$= e^{\beta J' \sigma_1 \sigma_3}$



[illegible] ⇒ [illegible] ⇒ [illegible]

YCP

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$Z = \int D\phi\, e^{iS[\phi]}$ $\quad \phi = \underset{|k|\le b\Lambda}{\Phi} + \underset{b\Lambda\le|k|\le\Lambda}{\sigma}$ $\quad \cancel{|k|\le b\Lambda \le |k| \le \Lambda}$

$= \int D\Phi D\sigma\, e^{iS[\Phi,\sigma]} = \int D\Phi\, e^{iS[\Phi]} \int D\sigma\, e^{i\delta S[\Phi,\sigma]}$

$= \int_{|k|\le b\Lambda} D\Phi\, e^{iS_{eff}[\Phi]}$ $\quad$ re-scale . $|k'| \le b\Lambda$. $k' = b\cdot q$ $\quad |q| \le \Lambda$.

$\phi(k') = \underline{\phi(bq) = \xi\,\phi(q)} \rightarrow$ rescale

$\downarrow$

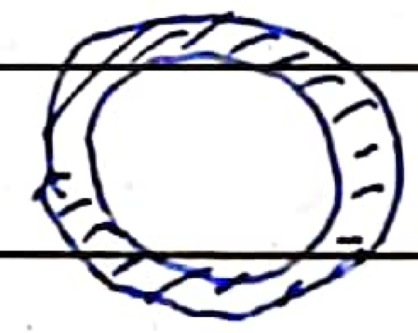
$\int_{|q|\le\Lambda} D\Phi\, e^{iS_{eff}[\Phi]}$ $\quad$ 本质: [illegible]

假设 $b>1$ → perturbation

类比. RG与Ising模型之对称性的变换. 二者比较

Model $H = -J\sum_i \sigma_i\sigma_{i+1}$. 实空间 | 动量空间 $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$

Treve number (- - .)



$\int D\sigma_{shell}$ 积分掉 $\quad \sigma: b\Lambda \le |k| \le \Lambda$

$H' = -J'\sum \sigma_{2i-1}\sigma_{2i+1}$

$\int D\Phi\, e^{iS_{eff}[\Phi]}$

R

Rescale 重新定index $\quad 2i-1 \to i \quad 2i+1 \to i+1$

$H = -J'\sum_i \sigma_i\sigma_{i+1}$

odd → even

$k \to bq \quad |k'| \le b\Lambda \quad |q| \le \Lambda$

$\phi(k') = \xi\,\phi(q) \quad |q| \le \Lambda$

找不动点 $\qquad$ 找不动点(临界)

若 $J' = J$ 对称性的变换不变 $\qquad$ 找对称性的变换 action

$S_{eff}[\Phi] = S[\phi]$

$\downarrow$

不动点 $R(J) = J'$ $\qquad R(R(J))$

R 是不可逆的

$\downarrow$

不动点 $R(J) = J$ $\qquad J \underset{\not\leftarrow}{\xrightarrow{R}} J' \underset{\not\leftarrow}{\xrightarrow{R}} J''$

YCP

$\delta J' = (\quad)\,\delta J$ $\qquad \delta J' + J^* = R(J^* + \delta J) = R(J^*) + \frac{\partial R}{\partial J^*}\,\delta J$

$\delta J' = \left(\frac{\partial R}{\partial J^*}\right)\cdot \delta J$