2D $\delta(x)$势.

$\lambda \sum_k \frac{1}{k^2+E} = 1$. bound state $-E$.

or $\frac{1}{\lambda} = \sum_k \frac{1}{k^2+E}$.

$k \le |\Lambda|$, 则有 $\lambda = \lambda(\Lambda)$.

$$\begin{cases} \frac{1}{\lambda_R} = \sum_k \frac{1}{k^2+\mu},\ \mu \to \mu_R \\ \frac{1}{\lambda} = \sum_k \frac{1}{k^2+E},\ \lambda \to \lambda_R \end{cases}$$

Bare perturbation theory.
方法简单.

重整化.

$\frac{1}{\lambda} - \frac{1}{\lambda_R} \equiv \sum_k \left(\frac{1}{k^2+E} - \frac{1}{k^2+\mu}\right)$

抵消项.

$\frac{1}{\lambda_R} = \sum_k \frac{1}{k^2+\mu}$.

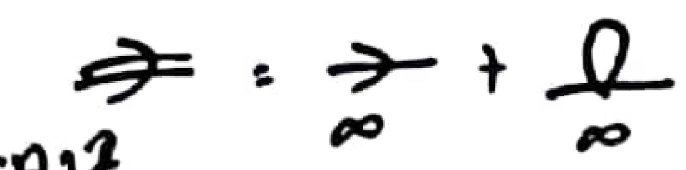
Renormalized Perturbation theory.

物理图像清晰.

ϕ^4. Theory. 唯微扰项.

$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$.

$m \to \infty$. m非物理量. bare Lagrange $\Lambda, m_\Lambda, \lambda_\Lambda$

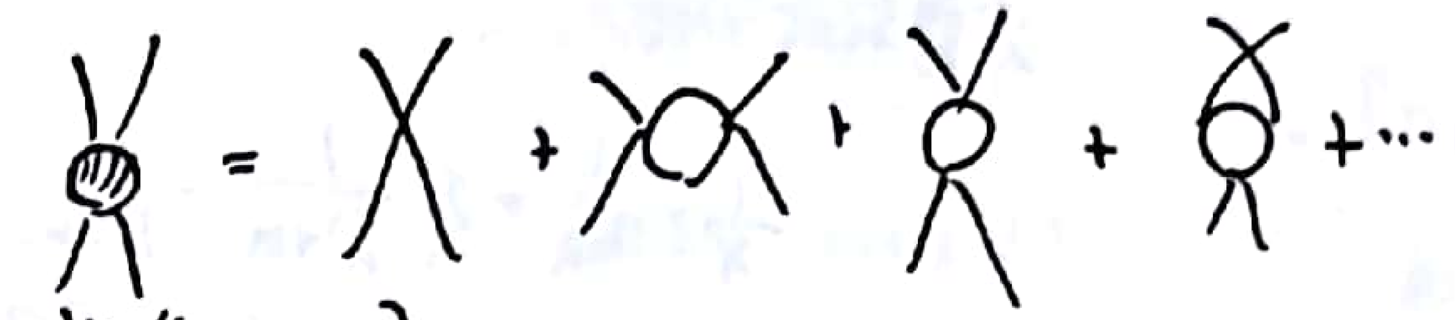


$= \infty + \infty$

浅层.

$\frac{1}{k^2-m_R^2} = \frac{1}{k^2-m^2} + \frac{1}{k^2-m^2}\Sigma(k,m)\frac{1}{k^2-m^2} + \dots \sim \frac{Z}{k^2-m_R^2}$.

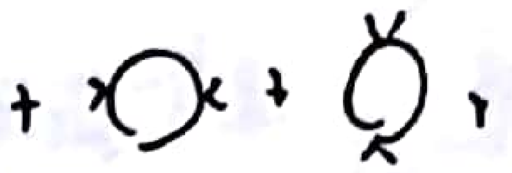
$\Rightarrow m_R^2 = m^2 + \Sigma(k,m^2)$, $\Sigma = m^2 - m_R^2$



λ

重整化后
可观测量.

工作量. $\mathcal{L}_0$.

$\lambda_R = \lambda +$  $, \dots$

$\propto \sum_k \left(\frac{1}{k^2+m^2}\right)\left(\frac{1}{k^2+m^2}\right) \sim \sum_k \left(\frac{1}{k^4}\right) \sim \ln\Lambda$.

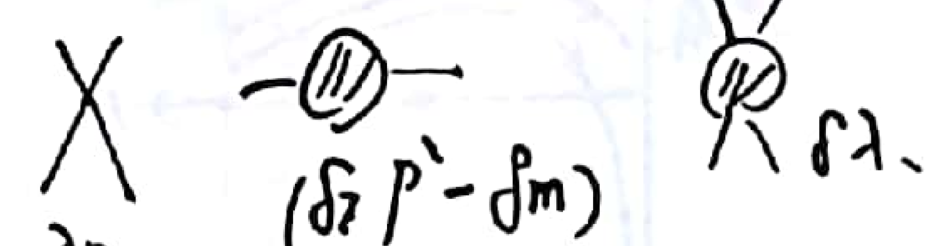
* 抵消项.

$\phi = \sqrt{Z}\phi_R$.

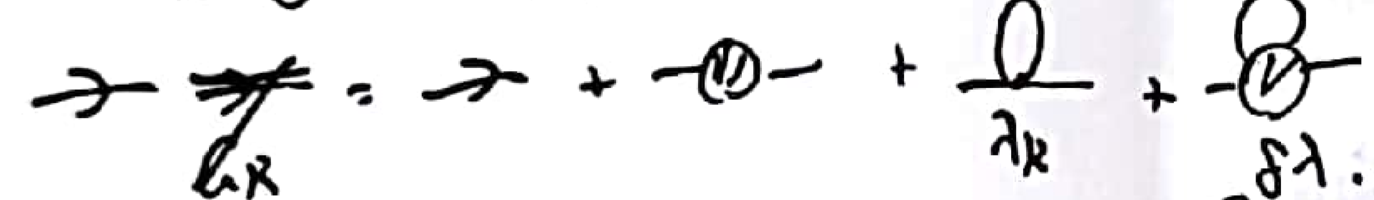
$\mathcal{L} = \mathcal{L}_R + \delta\mathcal{L}_c$

観测. 抵消项.

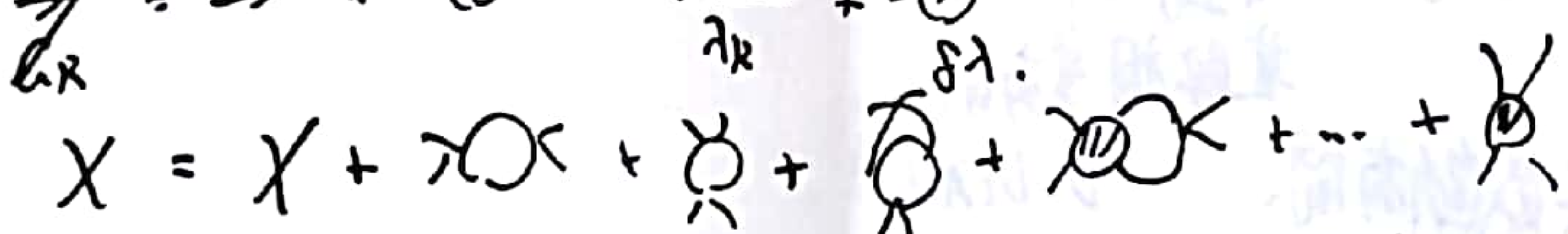
产生三项微扰.



λ_R $(\delta_Z p^2 - \delta m)$ $\delta\lambda$.



$\mathcal{L}_R$ λ_R $\delta\lambda$.



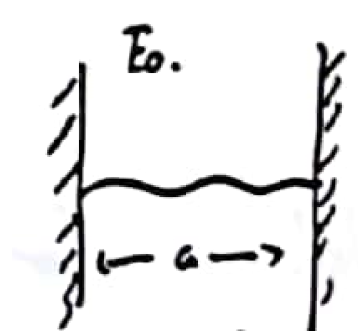
参数方差

$\lambda = \lambda_R + \delta\lambda$.

工作量 $\mathcal{L}_R$ 可観测, λ_R 已经调.

Casimir 力.

E_{ground} 发散.



E_0. $a = \frac{n}{2}\lambda$.

$E = pc = \frac{hc}{\lambda} = \frac{\hbar c\,n}{a}$.

$E_g = \frac{\hbar c}{2}\sum_{n=1}^{\infty}\left(\frac{n\pi}{a}\right)$. ~~$E_g = \frac{\hbar c}{2}\sum_{n=1}^{\infty}\left(\frac{n\pi}{a}\right)$~~ $\to$ 发散.

处理. $E_g \sim \frac{\hbar c}{2}\sum\left(\frac{n\pi}{a}\right)e^{-\frac{n}{\Lambda}}$ ~~处理. $E_g = \frac{\hbar c}{2}\sum\left(\frac{n\pi}{a}\right)e^{-\frac{n}{\Lambda}}$~~

能量发散
力有限.

致0条件 $= \infty \cdot \frac{\pi\hbar c}{24a}$.

$\frac{\partial E_g}{\partial a} = F = \frac{\pi\hbar c}{24a^2}$

(Zeta 正规化. $1+2+3+\dots+n = -\frac{1}{12}$)

重整化群. K. Wilson.
Renormalization Group.

臨界現象、應用於相變.

標度變換.

標度不變性. $f(\lambda x) = \lambda^{D} f(x)$.
參數改變, 性質不變.

例如分形. 自相似性.

$f(x) \sim \frac{1}{x^a}$.

$\langle S(x) S(0) \rangle \sim \frac{1}{x^a}$ (兩 spin).

$\langle S(\lambda x) S(\lambda 0) \rangle \sim \frac{1}{(\lambda x)^a} = \lambda^{-a} \left(\frac{1}{x^a}\right)$

*1. 广 Ising 模型、有效作用

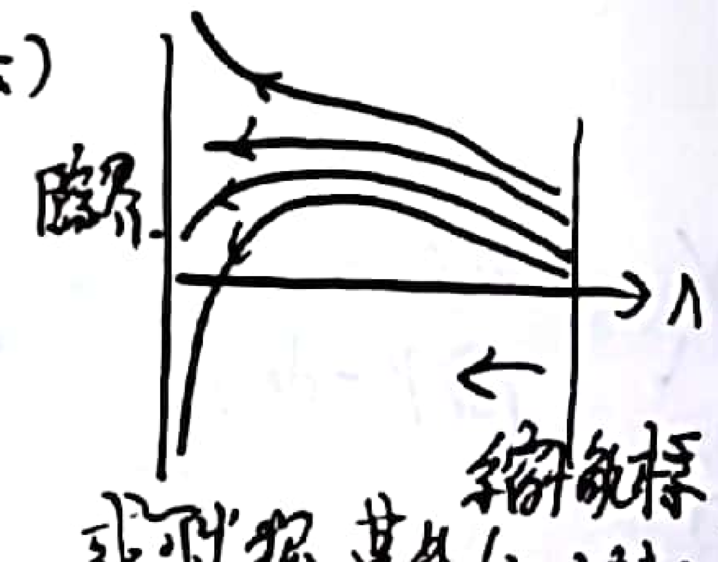
2. Callen–Symanzik equation.

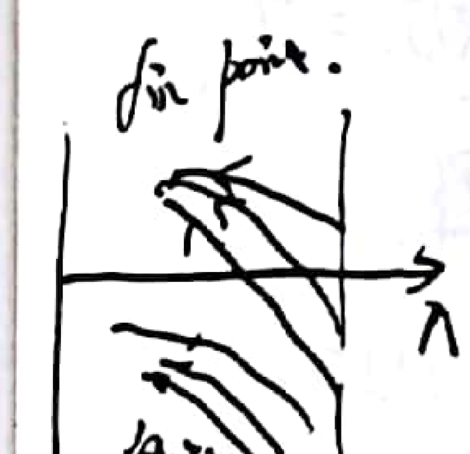
本課程無暇顧及.

$\frac{1}{\lambda} - \frac{1}{\lambda_R} = \sum \frac{1}{k^2+m} \cdot \frac{1}{k^2+\mu}$.

$\infty - \infty$ = 有限. 但不知過程如何.

慢慢縮小能標
觀察其過程.

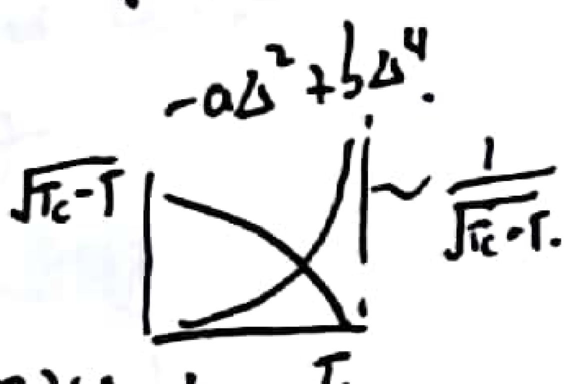




非可逆. 探其至 fix point.
其與相變有關.
$\nu = \nu(\lambda, \varepsilon)$

Landau 相變理論.

$-a\Delta^2 + b\Delta^4$.



臨界指數 $= \frac{1}{2}$.
指 $= \frac{1}{8}$
實驗、解析值

臨界指數是否與維度有關、(能標)

相似於重整化方法、重整化群適用範圍極廣.

$Z = \int D\phi\, e^{iS[\phi]} = \int D\bar{\phi}\, D\sigma\, e^{iS[\bar{\phi}, \sigma]} = \int D\bar{\phi}\, e^{iS[\bar{\phi}, \sigma]} \int D\sigma\, e^{i\Delta S[\bar{\phi}, \sigma]}$

$\phi = \bar{\phi} + \sigma$
$|k| \le b\Lambda$; $b\Lambda \le |k| \le \Lambda$.

$= \int D\bar{\phi}\, e^{iS_{eff}[\bar{\phi}]}$. 可微擾.
$|k| \le b\Lambda$.

$S[\phi, \sigma]$ 不一定與 $S_{eff}[\bar{\phi}]$ 相似.

Re-scale.

$|k'| \le b\Lambda$. $k' = bq$. $|q| \le \Lambda$.

$\phi(k') = \phi(bq) \Rightarrow$ rescale $= \xi\phi(q)$.

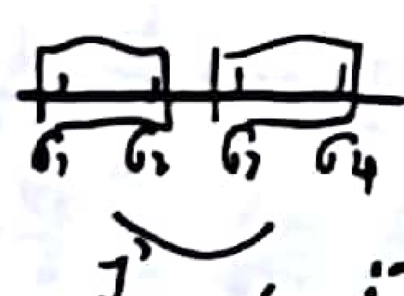
好處. ① 適用於任意模型.
②. $b \to 1$. 微擾僅看低階.

基本思想

① Ising model. for 1D. $Z = \mathrm{Tr}\, e^{-\beta H} = \mathrm{Tr}\, e^{\beta J (\sum_i \sigma_i \sigma_j)} = \mathrm{Tr}_{奇} \left(\mathrm{Tr}_{偶}\, e^{\beta J \sum_i \sigma_i \sigma_{i+1}} \right)$

$= \mathrm{Tr}_{奇}\, e^{\beta J' \sum_i \sigma_{2i-1} \sigma_{2i+1}}$

相似於以前的模型



$\sigma_1\ \sigma_2\ \sigma_3$.

$e^{\beta J (\sigma_1 \sigma_2 + \sigma_2 \sigma_3)} = e^{\beta J (\sigma_1 + \sigma_3)} + e^{-\beta J (\sigma_1 - \sigma_3)} = e^{\beta J' \sigma_1 \sigma_3}$
$\sigma_2 = +1$; $\sigma_2 = -1$.

用一個代替一個格子的行為

若仍遵守標度不變性.

實空間內 單元不斷擴大 (會失去細節)

失去短波近似 ~ 高能物理!

相對化,
損失短波信息.
扔掉高能物理.

逐漸看到長波信息

(Kardar 的)

大、(二) 動量空間內、將細節細節收入參數、考慮有效模型.

實空間

Model

$H=-J\sigma_i\sigma_{i+1}$.

Tr偶.

$H'=-J'$

$\sigma_{2i-1}\equiv\int\sigma_{2i-1}\sigma_{2i+1}$

重新標度 $\sigma_{2i-1}\Downarrow\sigma_i$

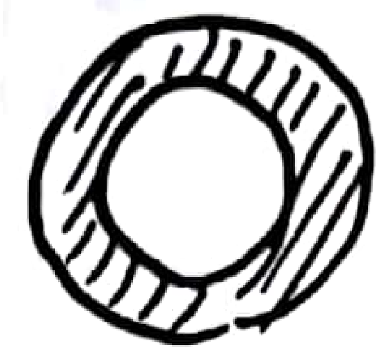
找 $J'=J$ → 標度不變點

標度不變點構成群.

k空間.

Model

$\mathcal{L}=\frac{1}{2}(\partial\phi)^2-\frac{m^2}{2}\phi^2-\frac{\lambda}{4!}\phi^4$.



$\int D\sigma_{shell}$ 積分掉.

or $\mathrm{Tr}_{b\Lambda<k<\Lambda}$.

$\int_{|k|<b\Lambda} D\phi\, e^{iS_{eff}[\phi]}$

$k'\to bq$. $|k'|<b\Lambda$. $|q|<\Lambda$.

ϕ代 $\phi(k')=Z\phi$

$S_{eff}[\phi]=S$. 標度不變作用量.

群

R.

$R(J)=J'$.

半群.

$R(R(J))=R(J')=J''$

但該過程是不可逆 由於細節已被拋掉

$J\xrightarrow{R}J'\xrightarrow{R}J''$

尋 $J^*=R(J^*)$.

$\delta J'+J^*$
$=R(J^*+\delta J)$
$=R(J^*)+\frac{\partial R}{\partial J^*}\cdot\delta J$.

$J'=J^*=\delta J$.

$\to\delta J'=(\quad)\delta J$.

穩定性.

總結.

兩種重正化等價性.(P326 Peskin).

重整化群. 自己的模型理解 / 粗粒化. 去細節過程. (a) 去高能過程 wilson.

$\therefore U\sim U(E,\Lambda)$.

臨界、標度不變.