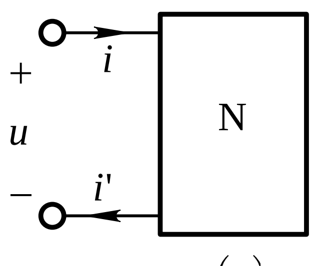
中国科学技术大学
University of Science and Technology of China

第14章 二端口网络

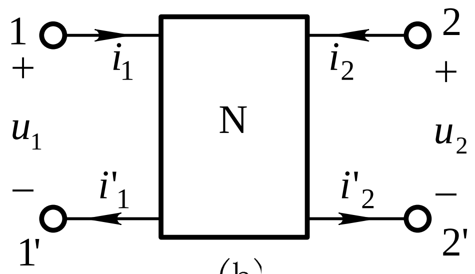
本章目录

§ 14.1 二端口网络

1 二端口网络：



(a)



(b)

端口变量：

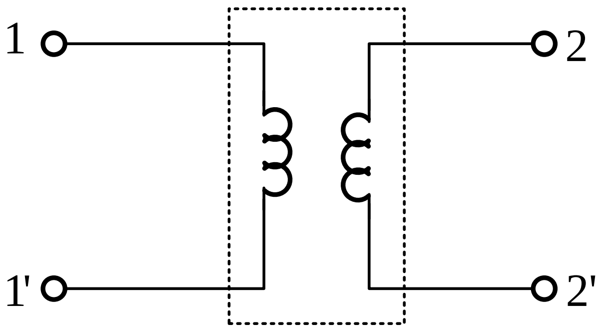
$U_1, I_1, U_2, I_2;$

$u_1, i_1, u_2, i_2;$

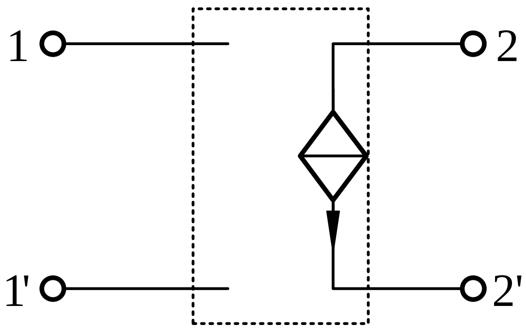
$\dot{U}_1, \dot{I}_1, \dot{U}_2, \dot{I}_2;$

$U_1(s), I_1(s), U_2(s), I_2(s)$

二端口网络举例



(a)



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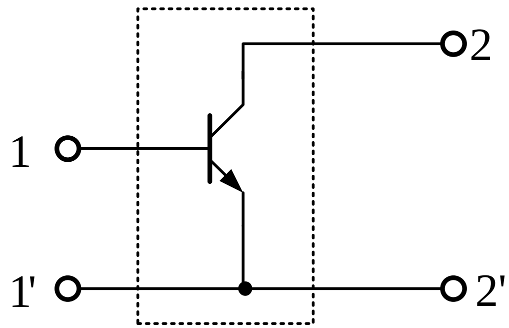
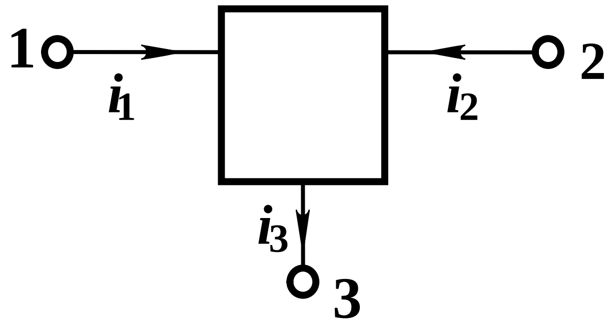


(c)

§ 14.1 二端口网络

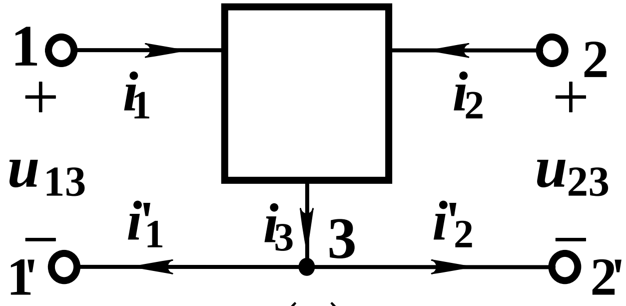
2 用二端口等效代替三端网络



(a)

$$i_3 = i_1 + i_2$$

$$u_{12} = u_{13} - u_{23}$$



(b)

三端网络只须用两个独立的端子电流和两个独立的端子间电压来描述如（**b**）

推广：一个n端网络可用n-1端口等效代替

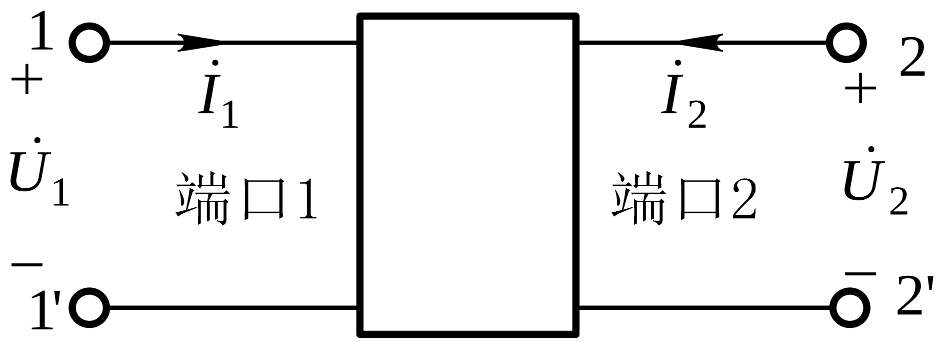
3 讨论范围

含线性***R***、***L***、***C***、***M***与线性受控源

不含独立源(复频域分析时，不包含附加电源)。

§ 14.2 短路导纳参数和开路阻抗参数

1 导纳参数方程



导纳参数方程或*Y*参数方程

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$

$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

矩阵形式： $\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$ 向量形式： $\dot{\boldsymbol{I}} = \boldsymbol{Y}\dot{\boldsymbol{U}}$

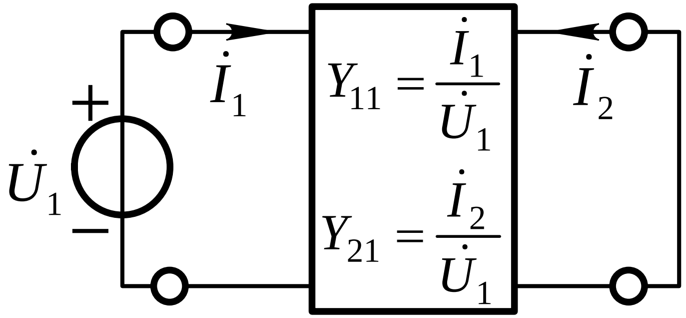
式中：$\dot{\boldsymbol{U}} = [\dot{U}_1, \dot{U}_2]^{\mathrm{T}}$ 和 $\dot{\boldsymbol{I}} = [\dot{I}_1, \dot{I}_2]^{\mathrm{T}}$ 分别表示端口电压和电流向量

$$\boldsymbol{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$ (导纳参数矩阵或***Y*** **参数矩阵)**

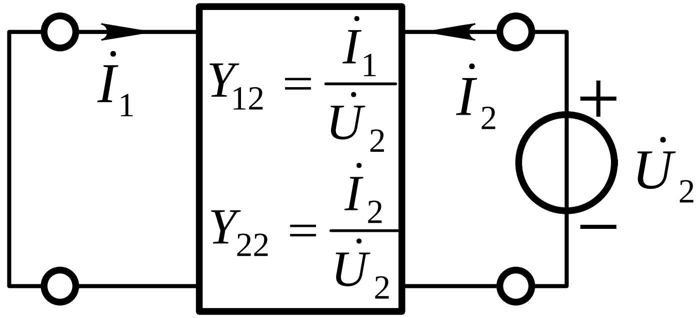
§ 14.2 短路导纳参数和开路阻抗参数

导纳参数的测定

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$
$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$



(a)



(b)

$$Y_{11} = \frac{\dot{I}_1}{\dot{U}_1}\bigg|_{\dot{U}_2=0} \qquad Y_{21} = \frac{\dot{I}_2}{\dot{U}_1}\bigg|_{\dot{U}_2=0} \qquad Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{U}_1=0} \qquad Y_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{U}_1=0}$$

***Y*参数也称为短路导纳参数**

Y_{11} --短路输入导纳

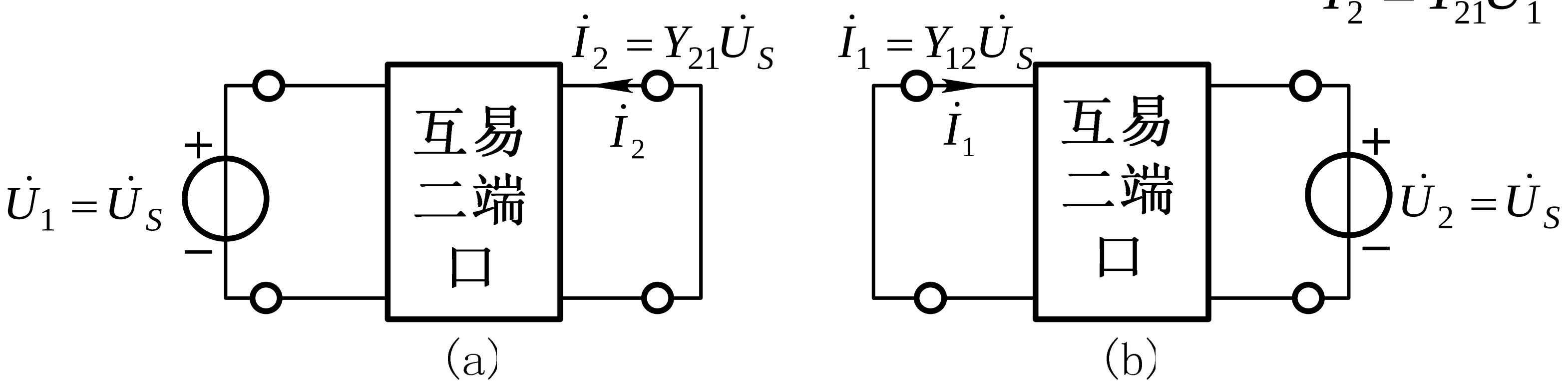
Y_{22}--短路输出导纳

Y_{12} Y_{21}--短路转移导纳

§ 14.2 短路导纳参数和开路阻抗参数

互易及对称情况：

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$
$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$



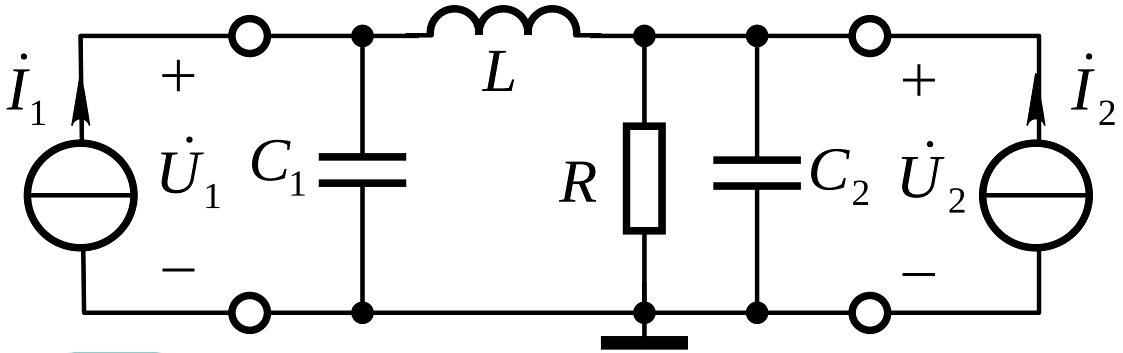
图中互易二端口的电流满足 $\dot{I}_2 = \dot{I}_1$ $\quad \dot{I}_1 = Y_{12}\dot{U}_S$ $\quad \dot{I}_2 = Y_{21}\dot{U}_S$

$$\Rightarrow \quad Y_{12} = Y_{21}$$

反之如果**Y** 参数满足 $Y_{12} = Y_{21}$ 则此二端口是互易二端口。

如果同时满足 $Y_{12} = Y_{21}$ 和 $Y_{11} = Y_{22}$ 则称为对称二端口。

§ 14.2 短路导纳参数和开路阻抗参数



例 **14.1** 求左图所示二端口**Y**参数矩阵。

解 用电流源置换两个端口列节点电压方程

$$\dot{I}_1 = (\mathrm{j}\omega C_1 + \frac{1}{\mathrm{j}\omega L})\dot{U}_1 - \frac{1}{\mathrm{j}\omega L}\dot{U}_2$$

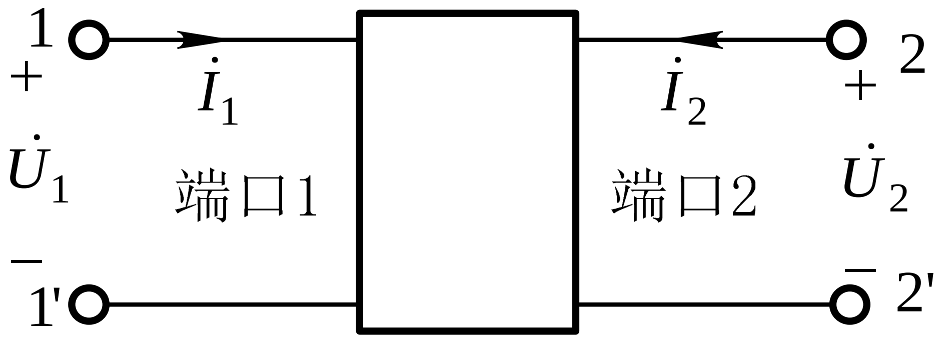
$$\dot{I}_2 = -\frac{1}{\mathrm{j}\omega L}\dot{U}_1 + (\frac{1}{R} + \mathrm{j}\omega C_2 + \frac{1}{\mathrm{j}\omega L})\dot{U}_2$$

上式的系数矩阵就是所求**Y** 参数矩阵：

$$\mathbf{Y} = \begin{bmatrix} \mathrm{j}(\omega C_1 - \frac{1}{\omega L}) & -\frac{1}{\mathrm{j}\omega L} \\ -\frac{1}{\mathrm{j}\omega L} & \frac{1}{R} + \mathrm{j}(\omega C_2 - \frac{1}{\omega L}) \end{bmatrix}$$

§ 14.2 短路导纳参数和开路阻抗参数

2 阻抗参数方程



阻抗参数方程或*Z*参数方程

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

矩阵形式：$\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix} = \begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}$

$\boldsymbol{Z} = \begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}$ 二端口阻抗参数矩阵或***Z*** 参数矩阵

Z 和 ***Y*** 互为逆矩阵

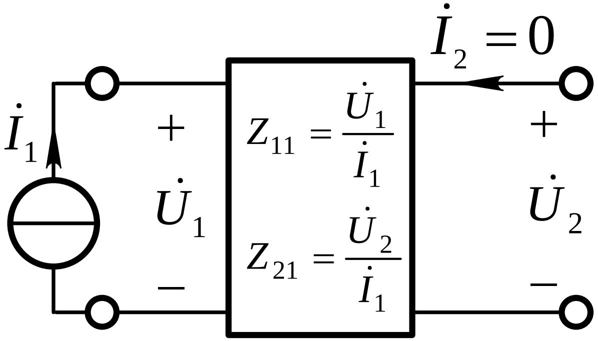
$\boldsymbol{Z} = \boldsymbol{Y}^{-1}$或$\boldsymbol{Y} = \boldsymbol{Z}^{-1}$

- 互易条件：$Z_{12} = Z_{21}$
- 对称条件：$Z_{12} = Z_{21}$和 $Z_{11} = Z_{22}$

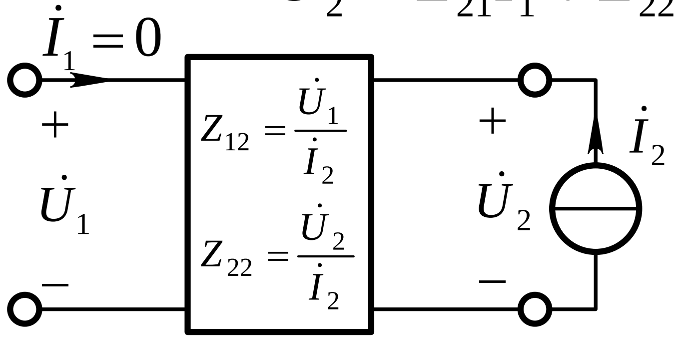
§ 14.2 短路导纳参数和开路阻抗参数

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

阻抗参数的测定：



(a)



(b)

$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} \quad Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0} \quad Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0} \quad Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

Z参数也称为开路阻抗参数

Z_{11} --开路输入阻抗

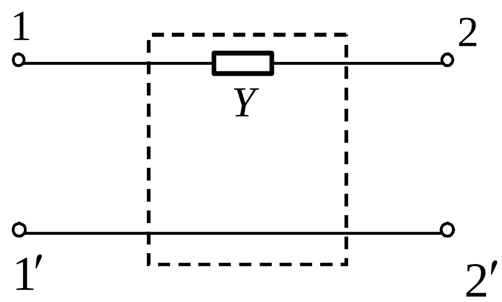
Z_{22}--开路输出阻抗

Z_{12} Z_{21}--开路转移阻抗

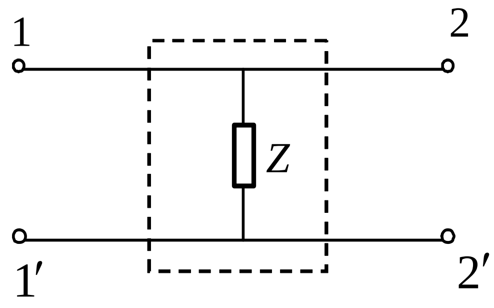
§ 14.2 短路导纳参数和开路阻抗参数

特殊情况

对于给定的二端口，有时不同时存在阻抗参数矩阵和导纳参数矩阵



简单串联元件组成的二端口



简单并联元件组成的二端口

图中没有阻抗参数矩阵，只有导纳参数矩阵

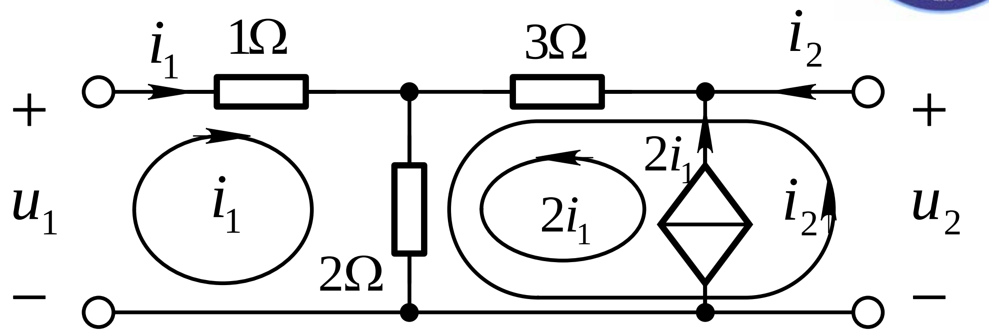
$$\boldsymbol{Y} = \begin{bmatrix} Y & -Y \\ -Y & Y \end{bmatrix}$$

图中没有导纳参数矩阵，只有阻抗参数矩阵：

$$\boldsymbol{Z} = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}$$

§ 14.2 短路导纳参数和开路阻抗参数

例**14.3** 求图所示二端口的阻抗参数矩阵。



解 选图中回路列回路电流方程

$$(1+2)\Omega\times i_1+2\Omega\times i_2+2\Omega\times 2i_1=u_1$$

$$2\Omega\times i_1+(2+3)\Omega\times 2i_1+(2+3)\Omega\times i_2=u_2$$

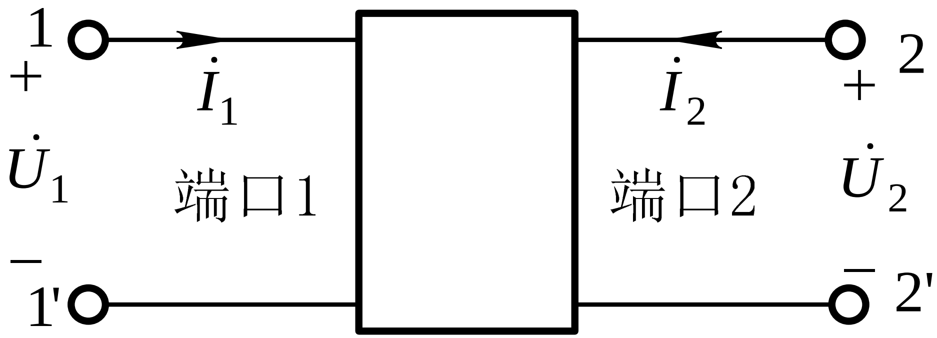
整理得： $7\Omega\times i_1+2\Omega\times i_2=u_1$

$$12\Omega\times i_1+5\Omega\times i_2=u_2$$

*Z*参数矩阵为 $\mathbf{Z}=\begin{bmatrix}7 & 2\\ 12 & 5\end{bmatrix}\Omega$

§ 14.3 传输参数和混合参数

1 传输参数方程



传输参数方程或*A*参数方程

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$
$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

矩阵形式：$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$

传输参数矩阵或 *A* 参数矩阵 $\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

互易性条件 $\Delta_A = A_{11}A_{22} - A_{12}A_{21} = 1$

对称条件 $\Delta_A = A_{11}A_{22} - A_{12}A_{21} = 1$ 及 $A_{11} = A_{22}$

§ 14.3 传输参数和混合参数

证明： 由：

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$

$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

解得：

$$\begin{cases} \dot{U}_1 = -\dfrac{Y_{22}}{Y_{21}}\dot{U}_2 + \dfrac{1}{Y_{21}}\dot{I}_2 \\ \dot{I}_1 = (Y_{12} - \dfrac{Y_{11}Y_{22}}{Y_{21}})\dot{U}_2 + \dfrac{Y_{11}}{Y_{21}}\dot{I}_2 \end{cases}$$

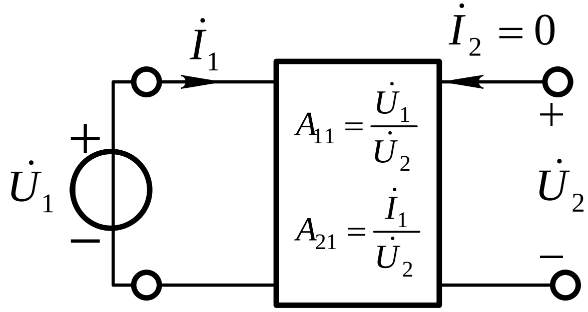
$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$

$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

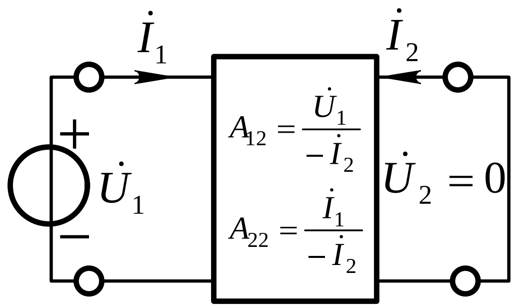
所以： $\Delta_A = A_{11}A_{22} - A_{12}A_{21} = \dfrac{Y_{22}Y_{11}}{Y_{21}^2} + \dfrac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}^2} = \dfrac{Y_{12}}{Y_{21}}$

§ 14.3 传输参数和混合参数

传输参数的测定：



(a)



(b)

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$
$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

$$A_{11}=\left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}\quad A_{21}=\left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}\quad A_{12}=\left.\frac{\dot{U}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}\quad A_{22}=\left.\frac{\dot{I}_1}{-\dot{I}_2}\right|_{\dot{U}_2=0}$$

逆传输参数方程

$$\left.\begin{aligned}\dot{U}_2 &= B_{11}\dot{U}_1 - B_{12}\dot{I}_1\\ \dot{I}_2 &= B_{21}\dot{U}_1 - B_{22}\dot{I}_1\end{aligned}\right\}$$

矩阵形式：

$$\begin{bmatrix}\dot{U}_2\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ -\dot{I}_1\end{bmatrix}$$

逆传输参数矩阵

$$\boldsymbol{B}=\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}$$

$$\boldsymbol{B}\neq\boldsymbol{A}^{-1}$$

§ 14.3 传输参数和混合参数

例**14.4** 求图**(a)**所示**T**形二端口网络的传输参数。

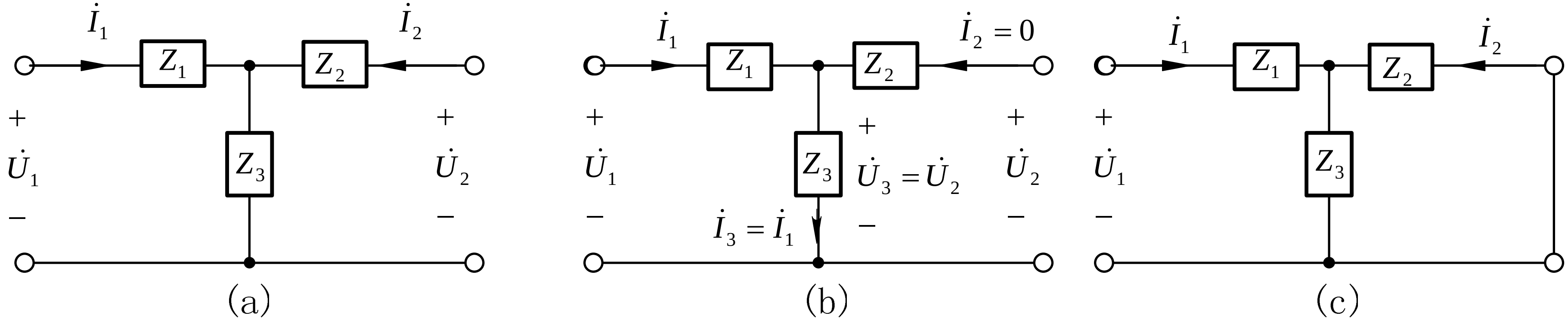


图14.16 例题14.4

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$
$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

解

(1) 令 $\dot{I}_2=0$ ，得图(b)由此求得 A_{11}、A_{21}

$$\dot{U}_2=\dot{U}_3=\frac{Z_3}{Z_1+Z_3}\dot{U}_1 \qquad A_{11}=\left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}=1+\frac{Z_1}{Z_3}$$

$$\dot{I}_1=\dot{I}_3=\frac{\dot{U}_2}{Z_3} \qquad A_{21}=\left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{I}_2=0}=\frac{1}{Z_3}$$

§ 14.3 传输参数和混合参数

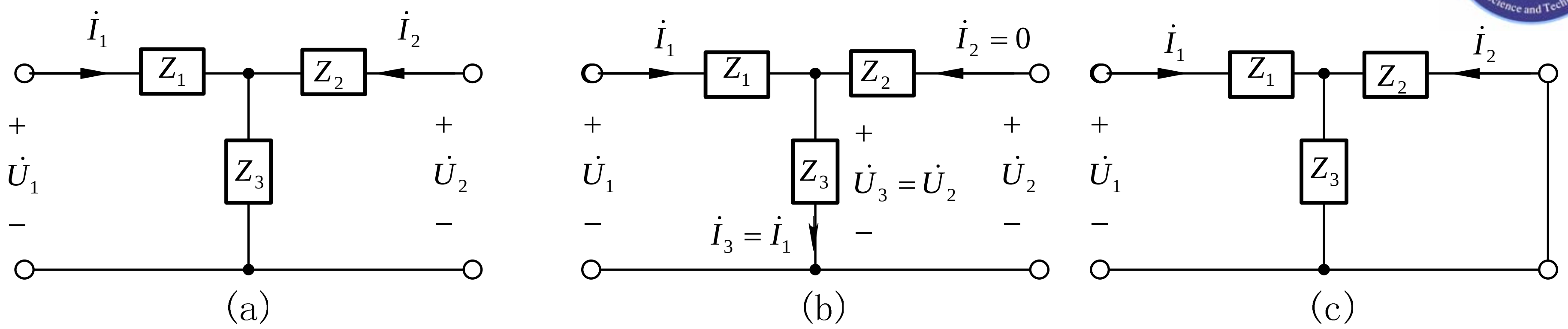


图14.16 例题14.4

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$
$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

(2) 再令 $\dot{U}_2=0$ 得图(c)又求得 A_{12}、A_{22}

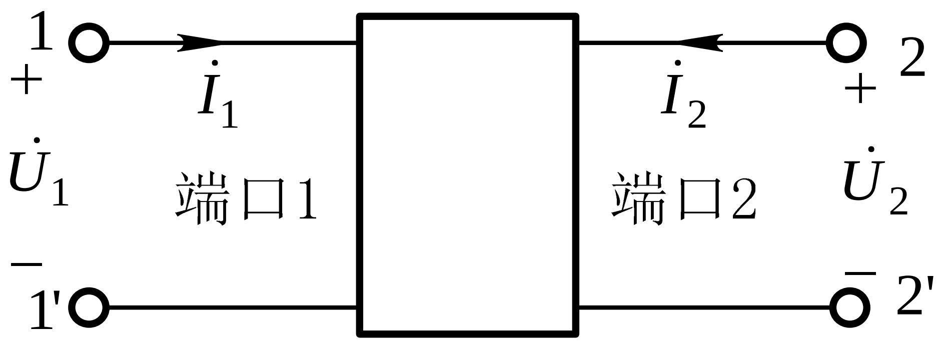
$$\dot{I}_2 = -\frac{Z_3}{Z_2+Z_3}\dot{I}_1 \qquad \dot{I}_1 = -(1+\frac{Z_2}{Z_3})\dot{I}_2 \qquad A_{22} = \frac{\dot{I}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0} = 1+\frac{Z_2}{Z_3}$$

$$\dot{U}_1 = Z_1\dot{I}_1 - Z_2\dot{I}_2 = -Z_1(1+\frac{Z_2}{Z_3})\dot{I}_2 - Z_2\dot{I}_2 = -(Z_1+Z_2+\frac{Z_1Z_2}{Z_3})\dot{I}_2$$

$$A_{12} = \frac{\dot{U}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0} = Z_1+Z_2+\frac{Z_1Z_2}{Z_3}$$

§ 14.3 传输参数和混合参数

2 混合参数方程



混合参数方程或 *H* 参数方程

$$\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$$
$$\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$$

矩阵形式：
$$\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix} = \begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}$$

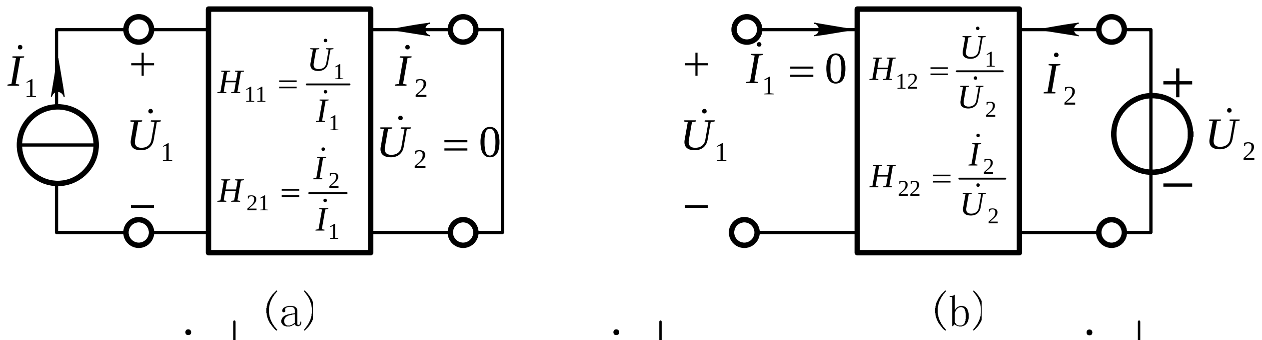
混合参数矩阵或 *H* 参数矩阵 $\mathbf{H} = \begin{bmatrix}H_{11} & H_{12}\\ H_{21} & H_{22}\end{bmatrix}$

互易性条件：$\boldsymbol{H_{12}=-H_{21}}$

对称条件：$\Delta_H = H_{11}H_{22} - H_{12}H_{21} = H_{11}H_{22} + H_{12}^2 = 1$

§ 14.3 传输参数和混合参数

H 参数测定：



$$\dot{U}_1 = H_{11}\dot{I}_1 + H_{12}\dot{U}_2$$
$$\dot{I}_2 = H_{21}\dot{I}_1 + H_{22}\dot{U}_2$$

$$H_{11}=\left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{U}_2=0} \quad H_{21}=\left.\frac{\dot{I}_2}{\dot{I}_1}\right|_{\dot{U}_2=0} \quad H_{12}=\left.\frac{\dot{U}_1}{\dot{U}_2}\right|_{\dot{I}_1=0} \quad H_{22}=\left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{I}_1=0}$$

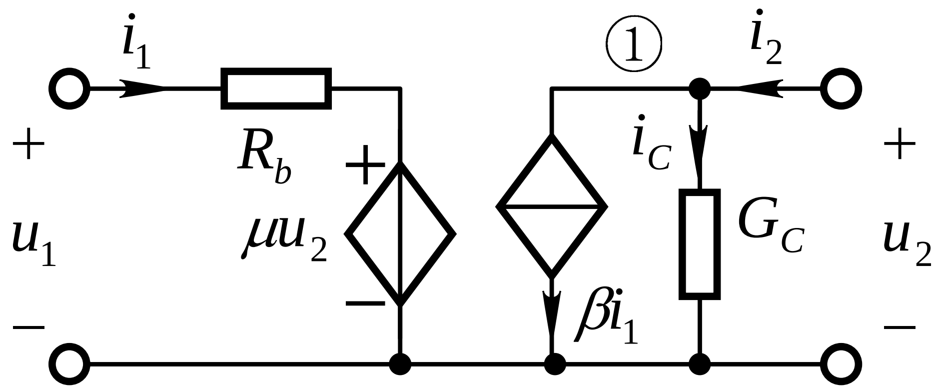
逆混合参数方程

$$\left.\begin{aligned}\dot{I}_1 &= G_{11}\dot{U}_1 + G_{12}\dot{I}_2 \\ \dot{U}_2 &= G_{21}\dot{U}_1 + G_{22}\dot{I}_2\end{aligned}\right\} \quad \begin{bmatrix}\dot{I}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{I}_2\end{bmatrix} \quad \mathbf{G}=\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\end{bmatrix}$$

G 和 **H** 互为逆矩阵　　$\mathbf{G}=\mathbf{H}^{-1}$　或　$\mathbf{H}=\mathbf{G}^{-1}$

§ 14.3 传输参数和混合参数

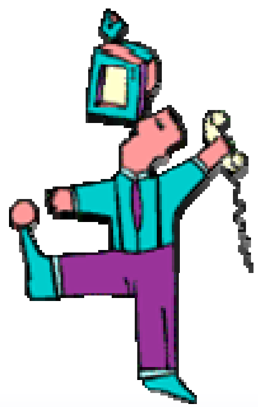
例**14.5**求图示半导体晶体管低频小信号等效电路的混合参数矩阵。



解 对输入端口所在回路列**KVL**方程 $u_1 = R_b i_1 + \mu u_2$

节点①列**KCL**方程： $i_2 = \beta i_1 + i_C = \beta i_1 + G_C u_2$

混合参数矩阵： $\mathbf{H} = \begin{bmatrix} R_b & \mu \\ \beta & G_C \end{bmatrix}$

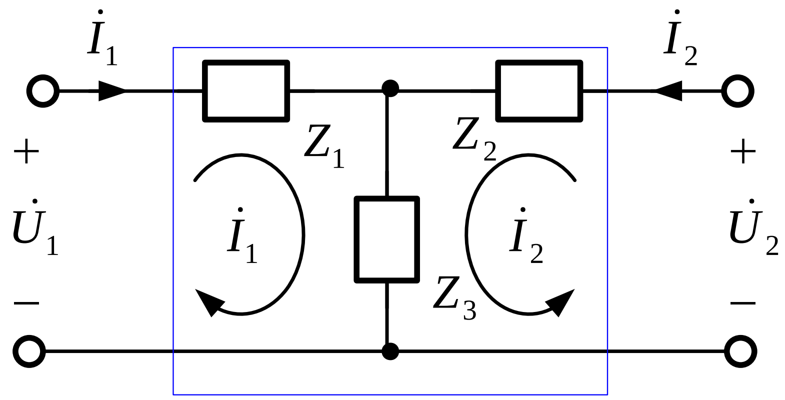


1. 互易二端口网络的等效电路

(1)给定Z参数宜选用T形等效电路

对图(a)列回路电流方程

$$\left.\begin{aligned}\dot{U}_1 &= (Z_1+Z_3)\dot{I}_1 + Z_3\dot{I}_2 \\ \dot{U}_2 &= Z_3\dot{I}_1 + (Z_2+Z_3)\dot{I}_2\end{aligned}\right\}$$



(a)

图14.20 二端口网络的T形等效电路

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

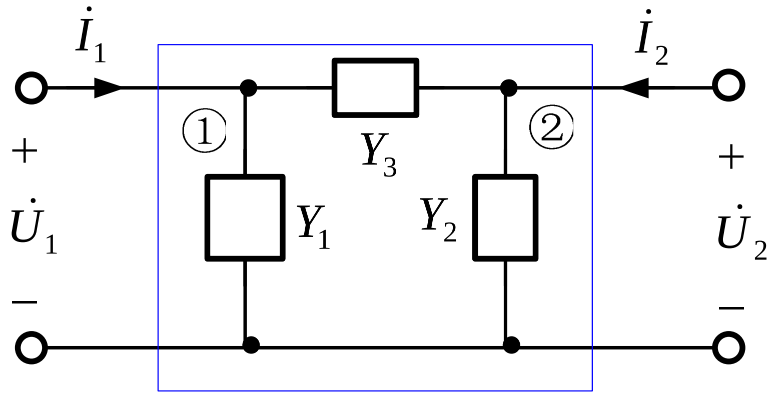
$$Z_{11} = Z_1 + Z_3 \qquad Z_{12} = Z_{21} = Z_3 \qquad Z_{22} = Z_2 + Z_3$$

$$\Rightarrow \quad Z_1 = Z_{11} - Z_{12} \qquad Z_2 = Z_{22} - Z_{12} \qquad Z_3 = Z_{12}$$

(2)给定Y参数宜选Π形等效电路

对图(b)列节点电压方程：

$$\left.\begin{aligned}\dot{I}_1 &= (Y_1+Y_3)\dot{U}_1 - Y_3\dot{U}_2 \\ \dot{I}_2 &= -Y_3\dot{U}_1 + (Y_2+Y_3)\dot{U}_2\end{aligned}\right\}$$



(b)

图14.20 二端口网络的Π形等效电路

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y_{11} = Y_1 + Y_3 \qquad Y_{12} = Y_{21} = -Y_3 \qquad Y_{22} = Y_2 + Y_3$$

$$\Rightarrow \quad Y_1 = Y_{11} + Y_{12} \qquad Y_2 = Y_{22} + Y_{21} \qquad Y_3 = -Y_{12}$$

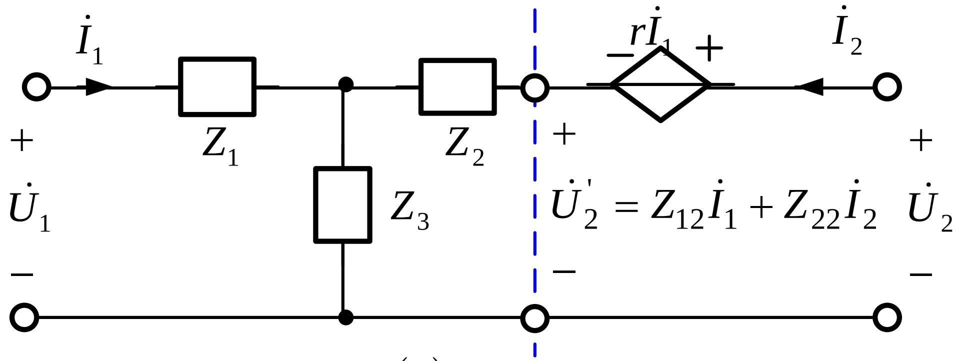
2. 非互易二端口网络的等效电路

(1)给定Z参数

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \qquad \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$

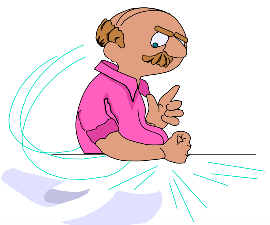
$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \qquad \dot{U}_2 = Z_{12}\dot{I}_1 + Z_{22}\dot{I}_2 + \left(Z_{21} - Z_{12}\right)\dot{I}_1$$

虚线左侧仍是一个互易性二端口的表达式，可用T形电路来代替；而在虚线右侧部分，则是一个电流控制电压源。



(a)

非互易性二端口的 T 形等效电路



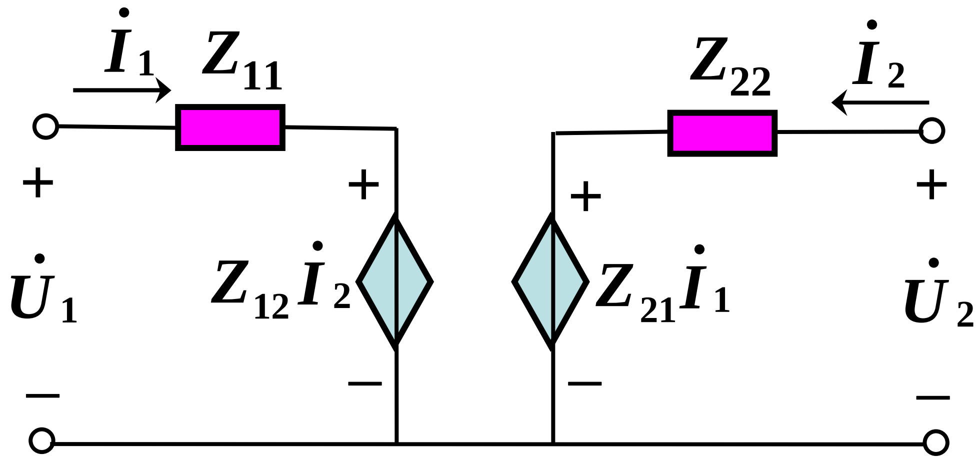
§ 14.4 二端口网络的等效电路

方法2：直接由参数方程得到等效电路。

$$\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2$$

$$\dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2$$

等效电路为：

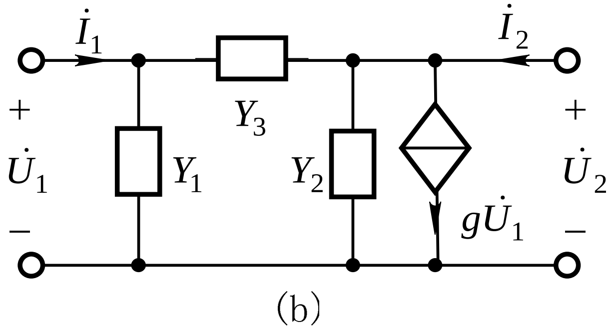


(2)给定Y参数

$$\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2$$

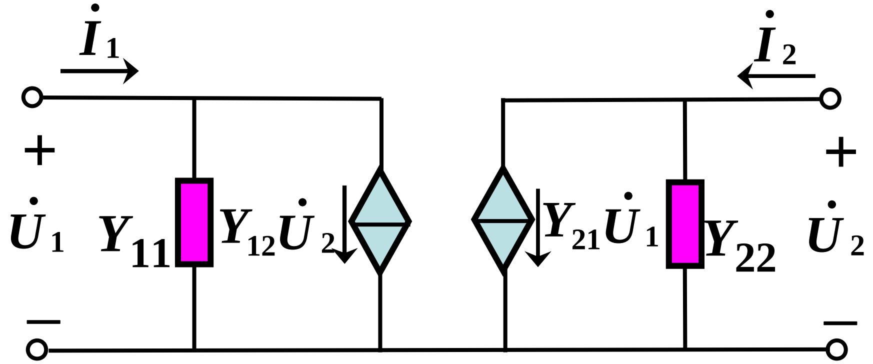
$$\dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2$$

$$\dot{I}_2 = Y_{12}\dot{U}_1 + Y_{22}\dot{U}_2 + (Y_{21} - Y_{12})\dot{U}_1$$



非互易性二端口的Π形等效电路

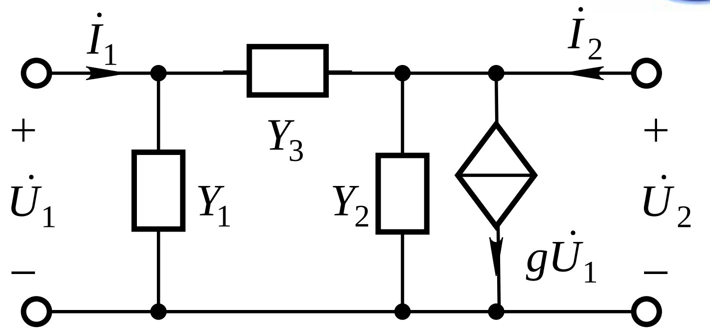
方法2：直接由参数方程得到等效电路。



§ 14.4 二端口网络的等效电路

例14.6设二端口网络复频域导纳参数矩阵为：

$$\mathbf{Y}=\begin{bmatrix}2s+1 & -1\\ -1.3 & 1+2/s\end{bmatrix}\text{(单位 S)}$$



(a)

求它的 ∏ 形等效电路参数。

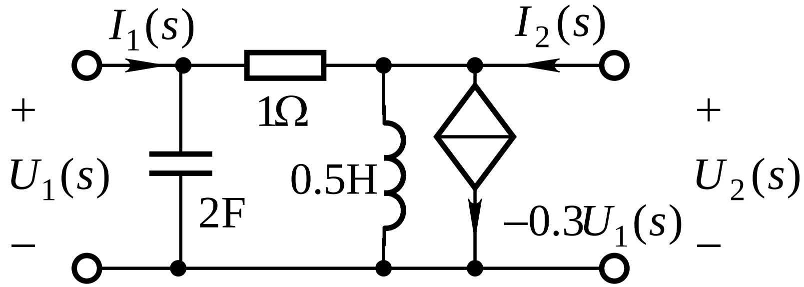
解 矩阵$\mathbf{Y}$ 中 $Y_{12}\neq Y_{21}$ ，应该用含有受控源的电路来等效，如图(a)所示。求得

$$Y_1(s)=Y_{11}(s)+Y_{12}(s)=2s$$

$$Y_2(s)=Y_{22}(s)+Y_{12}(s)=2/s$$

$$Y_3(s)=-Y_{12}(s)=1$$

$$g=Y_{21}(s)-Y_{12}(s)=-0.3$$



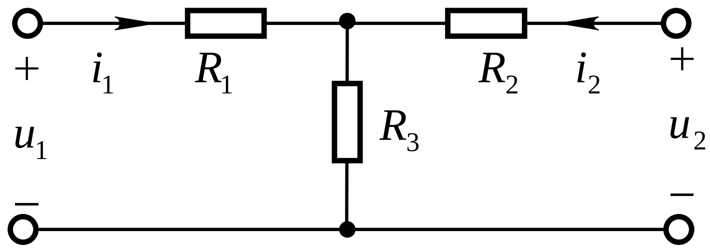
（b）

根据上述复频域中的导纳值可以得出对应的元件性质及参数值，如图(b)所示。

§ 14.4 二端口网络的等效电路

例14.7 已知二端口网络的传输参数矩阵为 $A=\begin{bmatrix}1.3 & 13.4\Omega \\ 0.1\text{S} & 1.8\end{bmatrix}$

求它的T形等效电路。



解 $\because A_{11}A_{22}-A_{12}A_{21}=1$

A满足互易条件，因此其T形等效电路不含受控源。

求**T**形电路的传输参数。根据例题**14.4**的计算结果

$$A_{11}=\left.\frac{\dot U_1}{\dot U_2}\right|_{\dot I_2=0}=1+\frac{Z_1}{Z_3}\qquad A_{21}=\left.\frac{\dot I_1}{\dot U_2}\right|_{\dot I_2=0}=\frac{1}{Z_3}\qquad A_{22}=\left.\frac{\dot I_1}{-\dot I_2}\right|_{\dot U_2=0}=1+\frac{Z_2}{Z_3}$$

$$\Longrightarrow\quad A_{11}=1+\frac{R_1}{R_3}=1.3\qquad A_{21}=\frac{1}{R_3}=0.1\text{S}\qquad A_{22}=1+\frac{R_2}{R_3}=1.8$$

由上述关系求得等效电路各电阻： $R_1=3\Omega$ $R_2=8\Omega$ $R_3=10\Omega$

§ 14.5 二端口网络与电源及负载的联接

1 二端口网络与电源和负载的联接

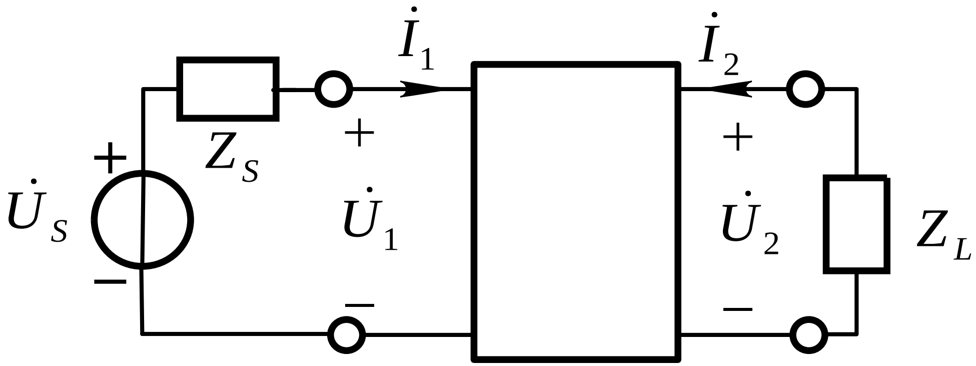


图14.24二端口网络与电源和负载的联接

约束端口的方程有：

(1)二端口参数方程

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$

$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

(2)电源支路方程

$$\dot{U}_1 = \dot{U}_S - Z_S\dot{I}_1$$

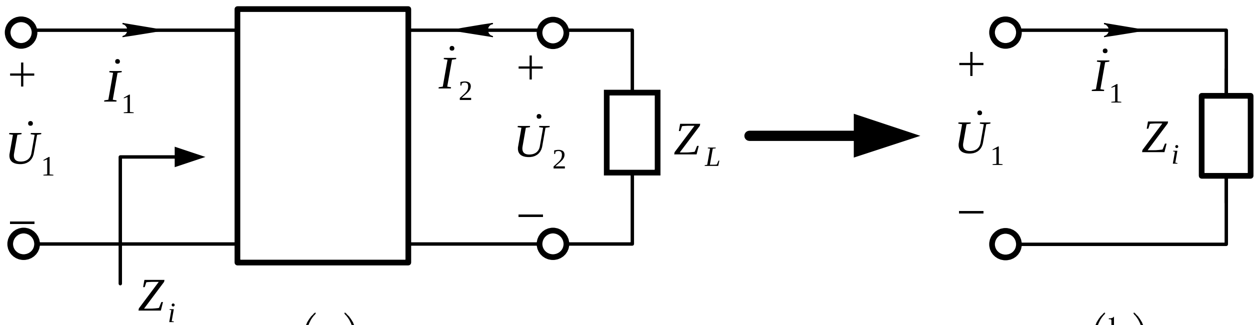
(3)负载支路方程

$$\dot{U}_2 = -Z_L\dot{I}_2$$

§ 14.5 二端口网络与电源及负载的联接

2 输入阻抗、输出阻抗

输入阻抗： 从输入端口向二端口视入，是一个线性无独立源一端口网络，可用一个阻抗来等效代替，称为输入阻抗。

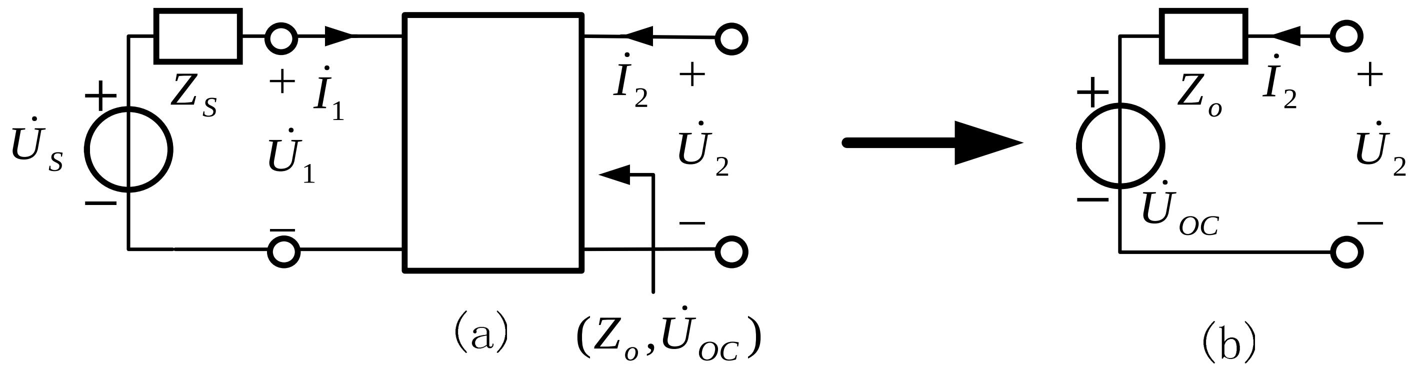


(a)　　　　(b)

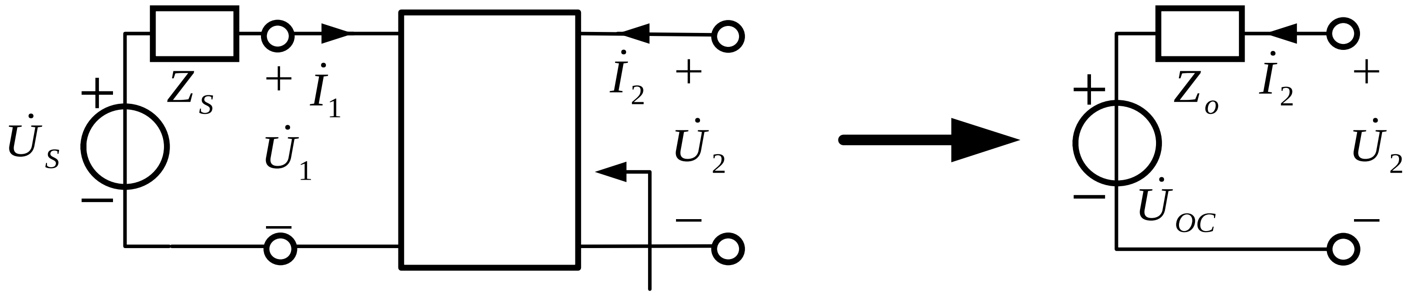
$$Z_\mathrm{i} = \frac{\dot{U}_1}{\dot{I}_1} = \frac{A_{11}\dot{U}_2 - A_{12}\dot{I}_2}{A_{21}\dot{U}_2 - A_{22}\dot{I}_2} = \frac{A_{11}(-Z_L\dot{I}_2) - A_{12}\dot{I}_2}{A_{21}(-Z_L\dot{I}_2) - A_{22}\dot{I}_2} = \frac{A_{11}Z_L + A_{12}}{A_{21}Z_L + A_{22}}$$

§ 14.5 二端口网络与电源及负载的联接

输出阻抗：从输出端口向二端口视入，是线性含独立源一端口网络，可用戴维南电路等效代替。从输出端口视入的等效阻抗，称为输出阻抗。



计算输出端口的戴维南等效电路：



(a)　$(Z_o, \dot{U}_{OC})$　　(b)

$$\left.\begin{aligned}\dot{U}_1 &= A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)\\ \dot{I}_1 &= A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)\\ \dot{U}_1 &= \dot{U}_S - Z_S\dot{I}_1\end{aligned}\right\} \Rightarrow A_{11}\dot{U}_2 - A_{12}\dot{I}_2 = \dot{U}_S - Z_S\dot{I}_1 = \dot{U}_S - Z_S(A_{21}\dot{U}_2 - A_{22}\dot{I}_2)$$

$$\Rightarrow \dot{U}_2 = \frac{\dot{U}_S}{A_{21}Z_S + A_{11}} + \frac{A_{22}Z_S + A_{12}}{A_{21}Z_S + A_{11}}\dot{I}_2 = \dot{U}_{OC} + Z_o\dot{I}_2$$

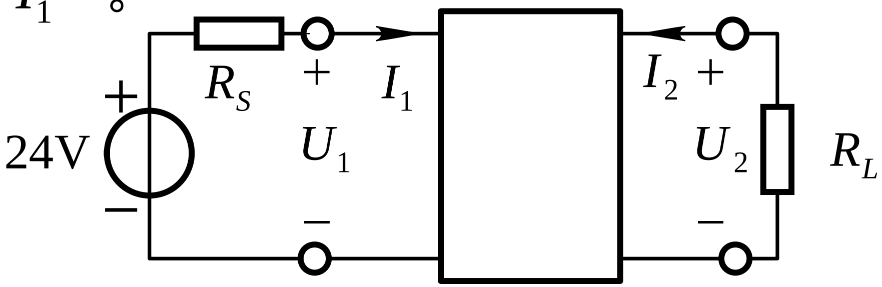
开路电压与等效输出阻抗

$$\dot{U}_{OC} = \frac{\dot{U}_S}{A_{21}Z_S + A_{11}} \qquad Z_o = \frac{\dot{U}_2}{\dot{I}_2}\bigg|_{\dot{U}_S=0} = \frac{A_{22}Z_S + A_{12}}{A_{21}Z_S + A_{11}}$$

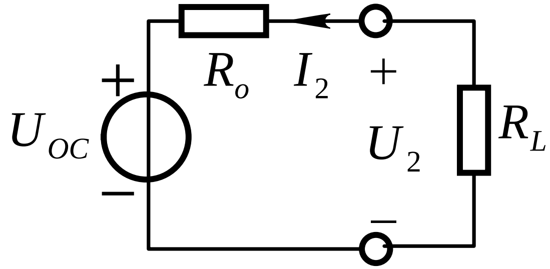
§ 14.5 二端口网络与电源及负载的联接

例**14.8**下图所示电路的传输参数矩阵为 $\mathbf{A}=\begin{bmatrix}1.5 & 5\Omega \\ 0.25\text{S} & 1.5\end{bmatrix}$ $R_S=2\Omega$

求负载电阻 R_L 消耗的最大功率以及此时输入端口的电压 U_1 和电流 I_1 。



(a)



(b)

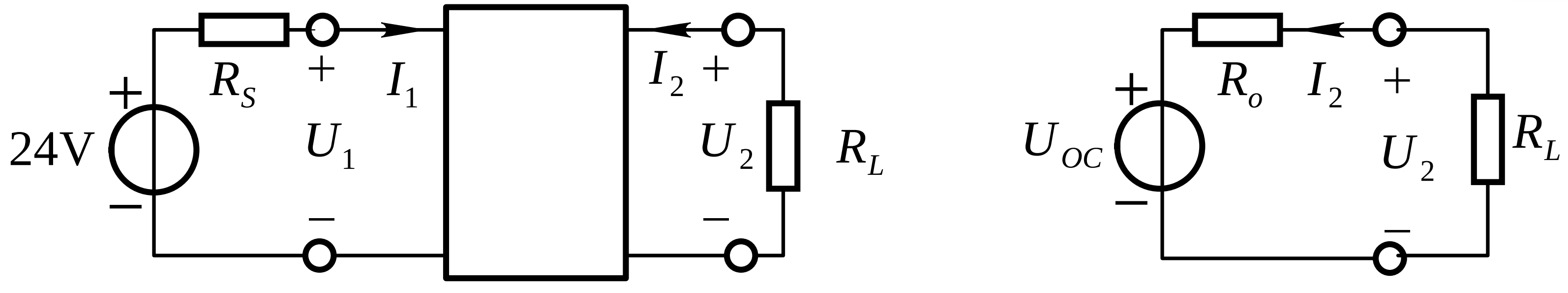
解

1) 求输出端口的戴维南等效电路

$$U_{OC}=\frac{U_S}{A_{21}R_S+A_{11}}=\frac{24\text{V}}{0.25\times 2+1.5}=12\text{V}$$

$$R_o=\frac{U_2}{I_2}=\frac{A_{22}R_S+A_{12}}{A_{21}R_S+A_{11}}=\frac{(1.5\times 2+5)\Omega}{0.25\times 2+1.5}=4\Omega$$

§ 14.5 二端口网络与电源及负载的联接



(a) (b)

2） 根据最大功率传输定理： $R_L = R_o = 4\Omega$ 时，吸收功率最大

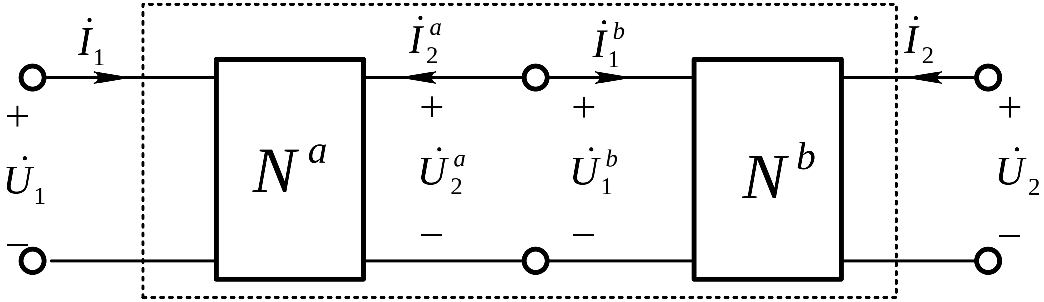
$$P_{\max} = \frac{U_{OC}^2}{4R_o} = \frac{(12\text{V})^2}{4\times 4\Omega} = 9\text{W}$$

$$U_2 = \frac{1}{2}U_{OC} = 6\text{V} \qquad I_2 = -\frac{U_{OC}}{2R_o} = -1.5\text{A}$$

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} 1.5 & 5\Omega \\ 0.25\text{S} & 1.5 \end{bmatrix} \begin{bmatrix} 6\text{V} \\ 1.5\text{A} \end{bmatrix} = \begin{bmatrix} 16.5\text{V} \\ 3.75\text{A} \end{bmatrix}$$

§ 14.6 二端口网络的级联

1 二端口网络的级联



已知：
$$\begin{bmatrix}\dot U_1\\ \dot I_1\end{bmatrix}=\begin{bmatrix}A_{11}^a & A_{12}^a\\ A_{21}^a & A_{22}^a\end{bmatrix}\begin{bmatrix}\dot U_2^a\\ -\dot I_2^a\end{bmatrix}$$

$$\begin{bmatrix}\dot U_1^b\\ \dot I_1^b\end{bmatrix}=\begin{bmatrix}A_{11}^b & A_{12}^b\\ A_{21}^b & A_{22}^b\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix}$$

N^a输出端口与 N^b输入端口存在 $\begin{bmatrix}\dot U_2^a\\ -\dot I_2^a\end{bmatrix}=\begin{bmatrix}\dot U_1^b\\ \dot I_1^b\end{bmatrix}$

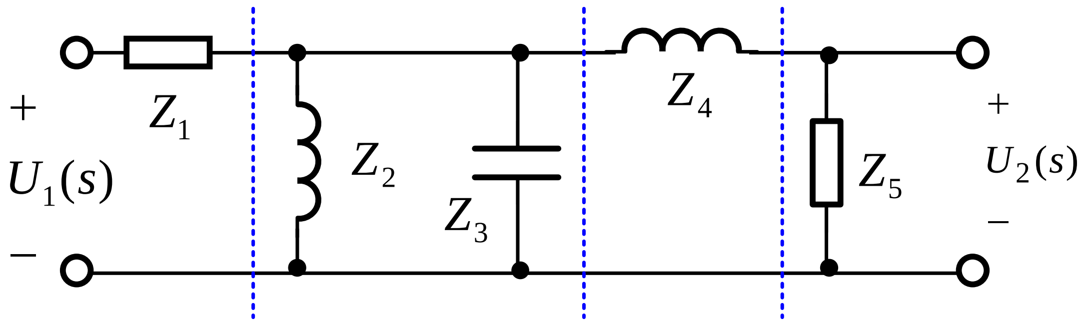
可得
$$\begin{bmatrix}\dot U_1\\ \dot I_1\end{bmatrix}=\begin{bmatrix}A_{11}^a & A_{12}^a\\ A_{21}^a & A_{22}^a\end{bmatrix}\begin{bmatrix}A_{11}^b & A_{12}^b\\ A_{21}^b & A_{22}^b\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix}$$

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}=\begin{bmatrix}A_{11}^a & A_{12}^a\\ A_{21}^a & A_{22}^a\end{bmatrix}\begin{bmatrix}A_{11}^b & A_{12}^b\\ A_{21}^b & A_{22}^b\end{bmatrix}$$

可推广到多个二端口级联的情况。注意，矩阵相乘的顺序应与级联顺序一致

§ 14.6 二端口网络的级联

例**14.9**求下图所示二端口的传输参数, 并求输出端口开路时转移函数 $U_2(s)/U_1(s)$. $Z_1(s)=1\Omega$, $Z_2(s)=2s$, $Z_3(s)=1/(3s)$, $Z_4(s)=4s$, $Z_5(s)=1$



解 将电路看成由四个二端口级联组成，各级的传输参数矩阵为

$$\begin{bmatrix} A_{11}^{(1)} & A_{12}^{(1)} \\ A_{21}^{(1)} & A_{22}^{(1)} \end{bmatrix}=\begin{bmatrix} 1 & Z_1(s) \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 1\Omega \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} A_{11}^{(2)} & A_{12}^{(2)} \\ A_{21}^{(2)} & A_{22}^{(2)} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ \dfrac{Z_2(s)+Z_3(s)}{Z_2(s)Z_3(s)} & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ \dfrac{6s^2+1}{2s} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{11}^{(3)} & A_{12}^{(3)} \\ A_{21}^{(3)} & A_{22}^{(3)} \end{bmatrix}=\begin{bmatrix} 1 & 4s \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} A_{11}^{(4)} & A_{12}^{(4)} \\ A_{21}^{(4)} & A_{22}^{(4)} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ \dfrac{1}{Z_5(s)} & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 1\mathrm{S} & 1 \end{bmatrix}$$

§ 14.6 二端口网络的级联

整个二端口网络传输参数矩阵为

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1\Omega \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{6s^2+1}{2s} & 1 \end{bmatrix} \begin{bmatrix} 1 & 4s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1\mathrm{S} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{24s^3+14s^2+8s+1}{2s} & (12s^2+4s+3) \\ \dfrac{24s^3+6s^2+6s+1}{2s} & 12s^2+3 \end{bmatrix}$$

没接负载时由二端口网络传输参数方程

$$\dot{U}_1 = A_{11}\dot{U}_2 + A_{12}(-\dot{I}_2)$$

$$\dot{I}_1 = A_{21}\dot{U}_2 + A_{22}(-\dot{I}_2)$$

可得：$U_1(s) = A_{11}U_2(s) - A_{12}I_2(s) = A_{11}U_2(s)$

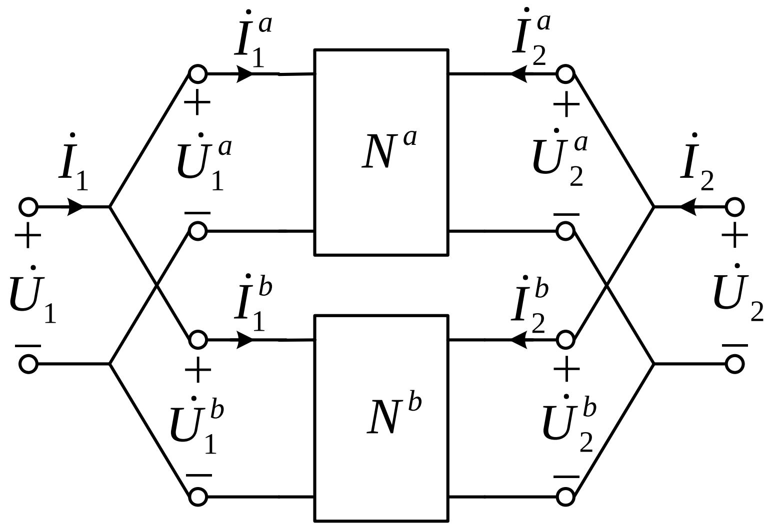
求得转移电压比为 $K_U(s) = \left.\dfrac{U_2(s)}{U_1(s)}\right|_{I_2=0} = \dfrac{1}{A_{11}} = \dfrac{2s}{24s^3+14s^2+8s+1}$

补充

2 二端口网络的并联

已知：$\begin{bmatrix}\dot{I}_1^a\\ \dot{I}_2^a\end{bmatrix}=\begin{bmatrix}Y_{11}^a & Y_{12}^a\\ Y_{21}^a & Y_{22}^a\end{bmatrix}\begin{bmatrix}\dot{U}_1^a\\ \dot{U}_2^a\end{bmatrix}$

$\begin{bmatrix}\dot{I}_1^b\\ \dot{I}_2^b\end{bmatrix}=\begin{bmatrix}Y_{11}^b & Y_{12}^b\\ Y_{21}^b & Y_{22}^b\end{bmatrix}\begin{bmatrix}\dot{U}_1^b\\ \dot{U}_2^b\end{bmatrix}$



可得：

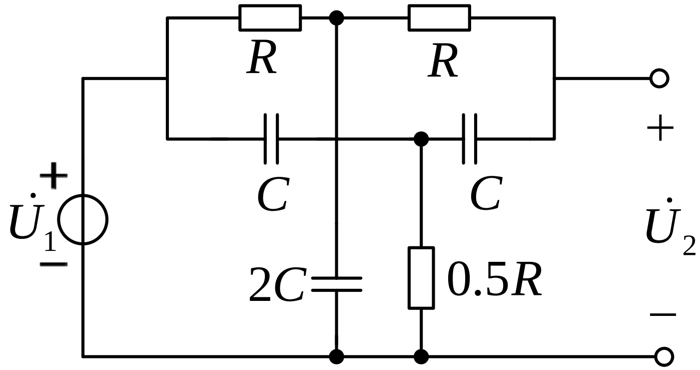
$$\begin{bmatrix}\dot{I}_1\\ \dot{I}_2\end{bmatrix}=\begin{bmatrix}\dot{I}_1^a+\dot{I}_1^b\\ \dot{I}_2^a+\dot{I}_2^b\end{bmatrix}=\begin{bmatrix}\dot{I}_1^a\\ \dot{I}_2^a\end{bmatrix}+\begin{bmatrix}\dot{I}_1^b\\ \dot{I}_2^b\end{bmatrix}=\begin{bmatrix}Y_{11}^a & Y_{12}^a\\ Y_{21}^a & Y_{22}^a\end{bmatrix}\begin{bmatrix}\dot{U}_1^a\\ \dot{U}_2^a\end{bmatrix}+\begin{bmatrix}Y_{11}^b & Y_{12}^b\\ Y_{21}^b & Y_{22}^b\end{bmatrix}\begin{bmatrix}\dot{U}_1^b\\ \dot{U}_2^b\end{bmatrix}$$

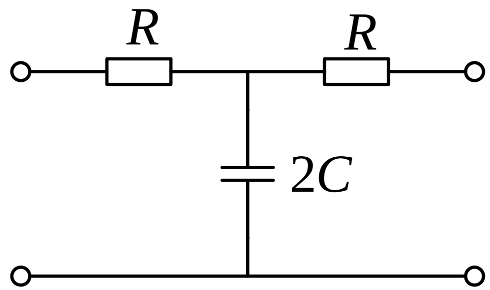
$$=\left\{\begin{bmatrix}Y_{11}^a & Y_{12}^a\\ Y_{21}^a & Y_{22}^a\end{bmatrix}+\begin{bmatrix}Y_{11}^b & Y_{12}^b\\ Y_{21}^b & Y_{22}^b\end{bmatrix}\right\}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}Y_{11}^a+Y_{11}^b & Y_{12}^a+Y_{12}^b\\ Y_{21}^a+Y_{21}^b & Y_{22}^a+Y_{22}^b\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1\\ \dot{U}_2\end{bmatrix}$$

注意：两个二端口网络并联时，其端口条件可能被破坏，此时上述关系式将不成立。

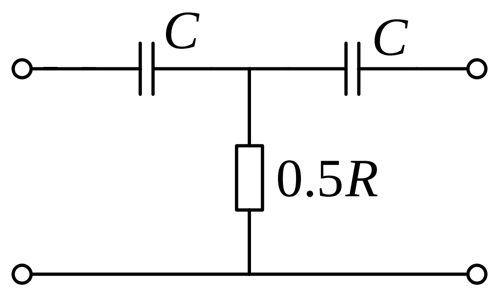
补充

例：习题6.16求$\dot{U}_2/\dot{U}_1$





图(a)



图(b)

由图(a) $Z^a=\begin{bmatrix} R+\dfrac{1}{\mathrm{j}2\omega C} & \dfrac{1}{\mathrm{j}2\omega C} \\ \dfrac{1}{\mathrm{j}2\omega C} & R+\dfrac{1}{\mathrm{j}2\omega C}\end{bmatrix}$

$$\therefore Y_{11}^a=Y_{22}^a=\frac{Z_{11}^a}{\Delta Z^a}=\frac{1+\mathrm{j}2\omega CR}{2R(1+\mathrm{j}\omega CR)}$$

$$Y_{12}^a=Y_{21}^a=-\frac{Z_{12}^a}{\Delta Z^a}=\frac{-1}{2R(1+\mathrm{j}\omega CR)}$$

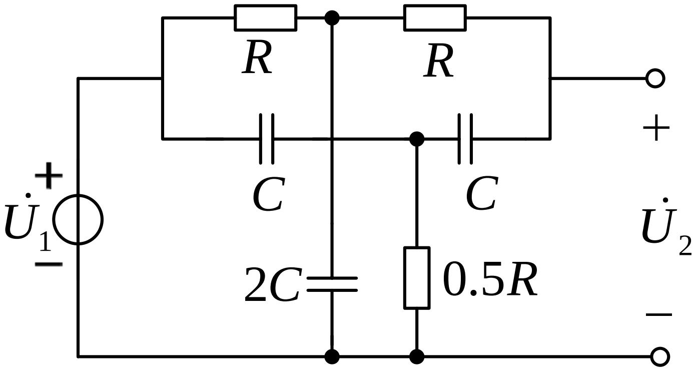
由图(b) $Z^b=\begin{bmatrix} 0.5R+\dfrac{1}{\mathrm{j}\omega C} & 0.5R \\ 0.5R & 0.5R+\dfrac{1}{\mathrm{j}\omega C}\end{bmatrix}$

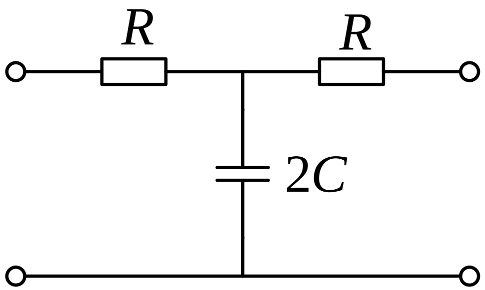
$$\therefore Y_{11}^b=Y_{22}^b=\frac{Z_{11}^b}{\Delta Z^b}=\frac{-\omega^2C^2R^2+\mathrm{j}2\omega CR}{2R(1+\mathrm{j}\omega CR)}$$

$$Y_{12}^b=Y_{21}^b=-\frac{Z_{12}^b}{\Delta Z^b}=\frac{\omega^2C^2R^2}{2R(1+\mathrm{j}\omega CR)}$$

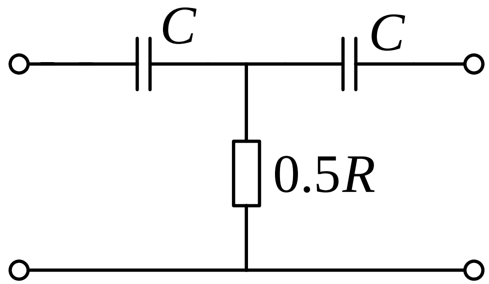
补充

例：习题6.16求$\dot{U}_2/\dot{U}_1$





图(a)



图(b)

$$Y_{11}^a=Y_{22}^a=\frac{Z_{11}^a}{\Delta Z^a}=\frac{1+\mathrm{j}2\omega CR}{2R(1+\mathrm{j}\omega CR)}\qquad Y_{11}^b=Y_{22}^b=\frac{Z_{11}^b}{\Delta Z^b}=\frac{-\omega^2C^2R^2+\mathrm{j}2\omega CR}{2R(1+\mathrm{j}\omega CR)}$$

$$Y_{12}^a=Y_{21}^a=-\frac{Z_{12}^a}{\Delta Z^a}=\frac{-1}{2R(1+\mathrm{j}\omega CR)}\qquad Y_{12}^b=Y_{21}^b=-\frac{Z_{12}^b}{\Delta Z^b}=\frac{\omega^2C^2R^2}{2R(1+\mathrm{j}\omega CR)}$$

复合二端口**Y**参数：$Y_{11}=Y_{22}=\dfrac{1-\omega^2C^2R^2+\mathrm{j}4\omega CR}{2R(1+\mathrm{j}\omega CR)}$　　$Y_{12}=Y_{21}=\dfrac{\omega^2C^2R^2-1}{2R(1+\mathrm{j}\omega CR)}$

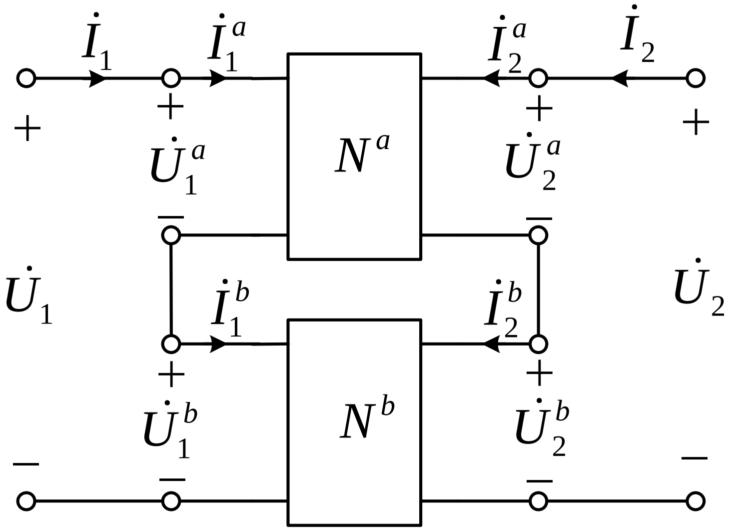
由**Y**参数方程：

$$\dot{I}_1=Y_{11}\dot{U}_1+Y_{12}\dot{U}_2$$
$$\dot{I}_2=Y_{21}\dot{U}_1+Y_{22}\dot{U}_2$$

$$\therefore\ \frac{\dot{U}_2}{\dot{U}_1}\bigg|_{\dot{I}_2=0}=-\frac{Y_{21}}{Y_{22}}=\frac{1-\omega^2C^2R^2}{1-\omega^2C^2R^2+\mathrm{j}4\omega CR}$$

补充

3 二端口网络的串联



已知：

$$\begin{bmatrix} \dot{U}_1^a \\ \dot{U}_2^a \end{bmatrix} = \begin{bmatrix} Z_{11}^a & Z_{12}^a \\ Z_{21}^a & Z_{22}^a \end{bmatrix} \begin{bmatrix} \dot{I}_1^a \\ \dot{I}_2^a \end{bmatrix}$$

$$\begin{bmatrix} \dot{U}_1^b \\ \dot{U}_2^b \end{bmatrix} = \begin{bmatrix} Z_{11}^b & Z_{12}^b \\ Z_{21}^b & Z_{22}^b \end{bmatrix} \begin{bmatrix} \dot{I}_1^b \\ \dot{I}_2^b \end{bmatrix}$$

可得：

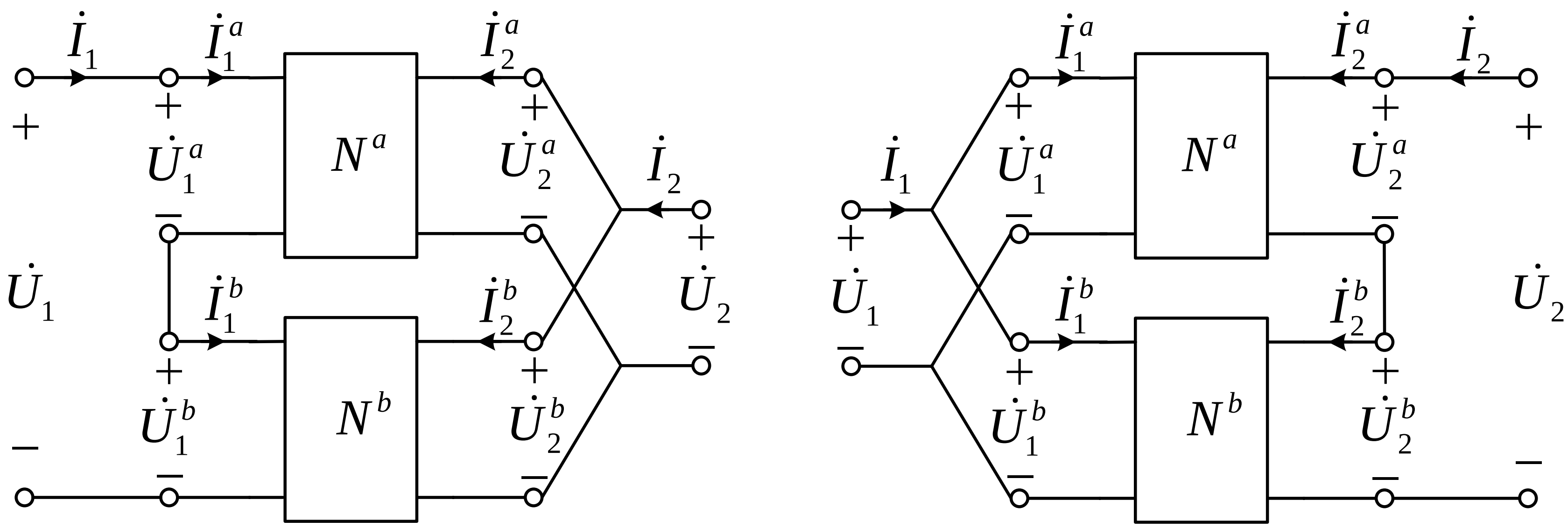
$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} \dot{U}_1^a + \dot{U}_1^b \\ \dot{U}_2^a + \dot{U}_2^b \end{bmatrix} = \begin{bmatrix} \dot{U}_1^a \\ \dot{U}_2^a \end{bmatrix} + \begin{bmatrix} \dot{U}_1^b \\ \dot{U}_2^b \end{bmatrix} = \begin{bmatrix} Z_{11}^a & Z_{12}^a \\ Z_{21}^a & Z_{22}^a \end{bmatrix} \begin{bmatrix} \dot{I}_1^a \\ \dot{I}_2^a \end{bmatrix} + \begin{bmatrix} Z_{11}^b & Z_{12}^b \\ Z_{21}^b & Z_{22}^b \end{bmatrix} \begin{bmatrix} \dot{I}_1^b \\ \dot{I}_2^b \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} Z_{11}^a & Z_{12}^a \\ Z_{21}^a & Z_{22}^a \end{bmatrix} + \begin{bmatrix} Z_{11}^b & Z_{12}^b \\ Z_{21}^b & Z_{22}^b \end{bmatrix} \right\} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Z_{11}^a + Z_{11}^b & Z_{12}^a + Z_{12}^b \\ Z_{21}^a + Z_{21}^b & Z_{22}^a + Z_{22}^b \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

注意：两个二端口网络串联时，其端口条件可能被破坏，此时上述关系式将不成立。

补充

4 二端口网络的串并联和并串联



二端口网络串并联　　　　二端口网络并串联

思考：二端口网络的串并联和并串联分别用哪种参数分析较方便？

小结

- 本章首先介绍二端口网络的各种参数方程及两种重要等效电路：**T**形和Π形等效电路；
- 其次介绍二端口网络的输入、输出阻抗等概念；
- 最后介绍二端口网络的级联规律。