- 证明

  $\sum_{k=0}^{n} x_k^j l_k(x) = x^j, j = 1,2,..,n$

  $\sum_{k=0}^{n}(x_k - x)^j l_k(x) = 0, j = 1,2,..,n$

- 设$x_j(j = 0,1,2,3,4,5)$为互异节点，$l_i(x)$为对应的5次插值基函数，计算
  $\sum_{i=0}^{5} x_i^5 l_i(0), \sum i = 05(x_i - x)^2 l_i(x)$和$\sum_{i=0}^{5}(x_i^5 + 2x_i^4 + x_i^3 + 1)l_i(x)$

- 设$f(x) \in C^1[a,b], x_0 \in (a,b)$,定义$f[x_0,x_0] = lim_{x->x_0}[x,x_0]$

  证明$f[x_0,x_0] = f'(x_0)$

- 设$f(x) = \frac{1}{a-x}$,且$a, x_0, x_1, \dots, x_n$互不相同，证明

  $f[x_0,x_1,\dots,x_k] = \frac{1}{(a-x_0)(a-x_1)\dots(a-x_k)}$

- 证明差商的莱布尼茨公式：若$p(x) = f(x)g(x)$则

  $p[x_0,x_1,\dots,x_k] = \sum_{k=0}^{n} f[x_0,x_1,\dots,x_k]g[x_k,x_{k+1},\dots,x_n]$

- 设要构造对数表$logx$,$1 \le x \le 10$，应如何选取步长，才能使分段线性插值误差小于10^{-6}

- 设$l_i(x)(i = 0,1,\dots,n)$是关于互异节点$x_i(i = 0,1,\dots n)$的拉格朗日基函数，求证

  $\sum_{i=0}^{n} l_i(0)x_i^{n+1} = (-1)^n x_0 x_1 \dots x_n$

- 设$f(x) = (x - x_0)(x - x_1)\dots(x - x_n), x_i$互异,求差商$f[x, x_0, \dots x_p]$

- 考虑下列插值问题：求一个二次多项式$p(x)$,使得$p(x_0) = y_0, p'(x_1) = m, p(x_2) = y_2$

  $x_0 \neq x_2$,y_0, m, y_2已知,试给出该问题具有唯一解的条件