第三次作业

- 数据表如下所示，试求满足下列数据和$S''(-2)=0, S''(2)=0$的三次样条插值函数，并计算$S(0)$的值。

x	**-2.00**	**-1.00**	**1.00**	**2.00**
$f(x)$	-4.00	3.00	5.00	12.00

  由题中数值计算得

  $h_0=1, h_1=2, h_2=1$

  $\lambda_1=\frac{2}{3}, \lambda_2=\frac{1}{3}$

  $\mu_1=\frac{1}{3}, \mu_2=\frac{2}{3}$

  $d_1=\frac{6}{-3}(\frac{-7}{-1}-\frac{-2}{-2})$

  $d_2=\frac{6}{-3}(\frac{-2}{-2}-\frac{-7}{-1})$

  由边界条件，$M_0=M_3=0$

$$\begin{bmatrix} 2 & \frac{2}{3} \\ \frac{2}{3} & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 2 \end{bmatrix}$$

  得到$M_1=-9, M_2=9$

  三次样条分段插值为

  $S(x)=10+7x-\frac{3}{2}(x+2)(x+1)(x+3) \ \ x\in[-2,-1]$

  $S(x)=4+x+\frac{3}{2}x(x+1)(x-1) \ \ x\in[-1,1]$

  $S(x)=-2+7x+\frac{3}{2}(x-2)(x-1)(3-x) \ \ x\in[1,2]$

- 数据表如下所示，试求满足下列数据和$S'(-1)=5, S'(3)=29$的三次样条插值函数，并计算$S(2)$的值。

x	**-1.00**	**0.00**	**1.00**	**3.00**
$f(x)$	2.00	3.00	4.00	29.00

  $h_0=1, h_1=1, h_2=2$

  $\lambda_1=\frac{1}{2}, \lambda_2=\frac{2}{3}$

  $\mu_1=\frac{1}{2}, \mu_2=\frac{1}{3}$

  $c_1=3(\frac{1}{2}+\frac{1}{2})=3$

  $c_2=3(\frac{2}{3}+\frac{1}{3}\frac{25}{2})=\frac{29}{2}$

  计算得$m_1=\frac{-17}{44}, m_2=\frac{28}{11}$

  $S_2(x)=x(x-3)^2+\frac{7}{11}(x-1)(x-3)^2+\frac{29}{4}(x-1)^2$

  $S_2(2)=\frac{435}{44}$

- 给定插值点$(x_i, f(x_i), f'(x_i))$，并定义序列$\{z_{2i}=z_{2i+1}=x_i, i=0,1,\ldots,n\}$

试证明差商型$Hermite$插值公式

$$H_{2n+1}(x)=f\left[z_0\right]+\sum_{k=1}^{2n+1} f\left[z_0, z_1, \cdots, z_k\right]\left(x-z_0\right)\left(x-z_1\right) \cdots\left(x-z_{k-1}\right)$$

$f[x,z_0]=\frac{f(x)-f(z_0)}{x-z_0}$

$f(x)=f(z_0)+(x-z_0)f[x,z_0]$

$f(x)=f(z_0)+f[z_0,z_1](x-z_0)+f[x,z_0,z_1](x-z_0)(x-z_1)$

不妨设当$n=p-1$时有$f(x)=H_{2p-1}(x)+R_{2p-1}(x)$

其中

$H_{2p-1}(x)=f(z_0)+\sum_{k=1}^{2p-1} f[z_0,z_1,\ldots,z_k](x-z_0)(x-z_1)\ldots(x-z_k)$

$R_{2p-1}(x)=f[x,z_0,\ldots,z_{2p-1}](x-z_0)(x-z_1)\ldots(x-z_{2p-1})$

当$n=p$时，

$f[x,z_0,\ldots,z_{2p}]=\frac{f[x,z_0,\ldots,z_{2p-1}]-f[z_0,\ldots,z_{2p}]}{x-z_{2p}}$

$f[x,z_0,\ldots,z_{2p+1}]=\frac{f[x,z_0,\ldots,z_{2p}]-f[z_0,\ldots,z_{2p+1}]}{x-z_{2p+1}}$

验证可得$f(x)=H_{2p+1}(x)+R_{2p+1}(x)$

即得所欲证。

- 推导三次样条函数插值在周期边界条件下的m和M关系式的边界条件

周期边界条件：

$S(x_0)=S(x_n), S'(x_0)=S'(x_n), S''(x_0)=S''(x_n)$

M关系式：

$M_0=M_n$(由$S''(x_0)=S''(x_n)$立刻得到)

由

$S''(x)=\frac{x_{i+1}-x}{h_i}M_i+\frac{x-x_i}{h_i}M_{i+1}$

$$\begin{aligned} S(x)=&\frac{\left(x_{i+1}-x\right)^3 M_i+\left(x-x_i\right)^3 M_{i+1}}{6 h_i}+\frac{\left(x_{i+1}-x\right) y_i+\left(x-x_i\right) y_{i+1}}{h_i} \\ &-\frac{h_i}{6}\left[\left(x_{i+1}-x\right) M_i+\left(x-x_i\right) M_{i+1}\right], \quad x \in\left[x_i, x_{i+1}\right] \end{aligned}$$

$S'(x_0)=\frac{-h_0}{2}M_0-\frac{y_0}{h_0}+\frac{y_1}{h_0}-\frac{h_0}{6}(-M_0+M_1)$

$S'(x_n)=\frac{h_{n-1}}{2}M_n-\frac{y_{n-1}}{h_{n-1}}+\frac{y_n}{h_{n-1}}-\frac{h_{n-1}}{6}(-M_{n-1}+M_n)$

得到

$\lambda_n M_1+\mu_n M_{n-1}+2M_n=d_n$

$d_n=\frac{6}{h_0+h_{n-1}}(f[x_0,x_1]-f[x_{n-1},x_n])$

$\lambda_0=\frac{h_0}{h_0+h_{n-1}}, M_n=1-\lambda_n$

m关系式

$m_0=m_n$

$\lambda_n m_{n-1}+2m_n+\mu_n m_1=c_n$

$\lambda_n=\frac{h_0}{h_0+h_{n-1}}$

$\mu_n=1-\lambda_n$

$c_n=3[\lambda_n f[x_{n-1},x_n]+\mu_n f[x_1,x_0]]$