Homework 3

Name Number

1,derive high order Tikhonov regularization $C=G^{\#}cov(d)(G^{\#})^{T}$,using $G=USV^{T}$

3. Consider the following problem in **cross-well tomography**. Two vertical wells are located 1600 m apart. A seismic source is inserted in one well at depths of 50, 150, . . . , 1550 m. A string of receivers is inserted in the other well at depths of 50 m, 150 m, . . . , 1550 m. See Figure 4.28. For each source-receiver pair, a travel time is recorded, with a measurement standard deviation of 0.5 ms. There are 256 ray paths and 256 corresponding data points. We wish to determine the velocity structure in the two-dimensional plane between the two wells.

Discretizing the problem into a 16 by 16 grid of 100 meter by 100 meter blocks gives 256 model parameters. The **G** matrix and noisy data, **d**, for this problem (assuming straight ray paths) are in the file **crosswell.mat**. The order of parameter indexing from the slowness grid to the model vector is row by row (e.g., Example 1.12).

a. Use the TSVD to solve this inverse problem using an L-curve. Plot the result.

b. Use zeroth-order Tikhonov regularization to solve this problem and plot your solution. Explain why it is difficult to use the discrepancy principle to select the regularization parameter. Use the L-curve criterion to select your regularization parameter. Plot the L-curve as well as your solution.

c. Use second-order Tikhonov regularization to solve this problem and plot your solution. Because this is a two-dimensional problem, you will need to implement a finite-difference approximation to the Laplacian (second derivative in the horizontal direction plus the second derivative in the vertical direction) in the roughening matrix. The **L** matrix can be generated using the following MATLAB code:

```
L=zeros(14*14,256);
k=1;
for i=2:15,
  for j=2:15,
    M=zeros(16,16);
    M(i,j)=-4;
    M(i,j+1)=1;
    M(i,j-1)=1;
    M(i+1,j)=1;
    M(i-1,j)=1;
    L(k,:)=reshape(M,256,1)';
    k=k+1;
  end
end
```

What, if any, problems did you have in using the L-curve criterion on this problem? Plot the L-curve as well as your solution.

d. Discuss your results. If vertical bands appeared in some of your solutions, can you explain why?