Solution 10.

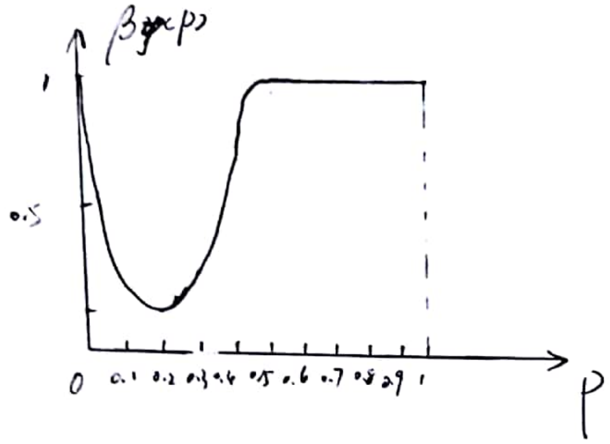
5.1 (1)

$$\beta_\varphi(p) = E_p(\varphi(X)) = 1 - P\left(2 \le \sum_{i=1}^{20} X_i \le 6\right) = 1 - \sum_{k=2}^{6} \binom{20}{k} p^k (1-p)^{20-k}$$

$\beta_\varphi(0) = 1$ $\quad \beta_\varphi(0.1) = 0.3942$ $\quad \beta_\varphi(0.2) = 0.1559$

$\beta_\varphi(0.3) = 0.3997$ $\quad \beta_\varphi(0.4) = 0.7506$ $\quad \beta_\varphi(0.5) = 0.9424$

$\beta_\varphi(0.6) = 0.9936$ $\quad \beta_\varphi(0.7) = 0.9997$ $\quad \beta_\varphi(0.8) = \beta_\varphi(0.9) = \beta_\varphi(1) = 1$



(2). $\sup\{\beta_\varphi(p), p \in \Theta_0\} = \beta_\varphi(0.2) = 0.1559$

∴ 真实水平 $\alpha = 0.1559$

检验水平可以为 $[0.559, 1]$ 的任意数.

$\beta_\varphi^*(0.05) = 1 - \beta_\varphi(0.05) = 0.2641$

□

5.2 (1) $H_0 = \{\mu_0\}$

$\Rightarrow \dfrac{\sqrt{n}(\bar{x}-\mu_0)}{3} \sim N(0,1)$

$P\left(\left|\dfrac{\sqrt{n}(\bar{x}-\mu_0)}{3}\right| \geq u_{0.025}\right) = 0.05$

即 $P\left(|\bar{x}-\mu_0| \geq \dfrac{3}{\sqrt{n}} u_{0.025}\right) = 0.05$

$\Rightarrow c = \dfrac{3}{\sqrt{n}} u_{0.025} = \dfrac{5.88}{\sqrt{n}}$

(2) $\beta(\mu) = E_\mu(\varphi(x))$ $\quad \bar{x} \sim N(\mu, \frac{9}{n})$

$= P(|\bar{x}-\mu_0| \geq c)$

$= P_\mu\left(\dfrac{\bar{x}-\mu_0}{3/\sqrt{n}} \geq \dfrac{c\sqrt{n}}{3}\right) + P_\mu\left(\dfrac{\bar{x}-\mu_0}{3/\sqrt{n}} \leq \dfrac{-c\sqrt{n}}{3}\right)$

$= P_\mu\left(\dfrac{\bar{x}-\mu}{3/\sqrt{n}} \geq \dfrac{\sqrt{n}}{3}(c+\mu_0-\mu)\right) + P_\mu\left(\dfrac{\bar{x}-\mu}{3/\sqrt{n}} \leq \dfrac{\sqrt{n}}{3}(-c+\mu_0-\mu)\right)$

$= 1 - \Phi\left(\dfrac{\sqrt{n}}{3}(c+\mu_0-\mu)\right) + \Phi\left(\dfrac{\sqrt{n}}{3}(-c+\mu_0-\mu)\right)$

(3) $n=25$ 时.

$\alpha = \beta(\mu_0) = 2\left[1-\Phi\left(\frac{5}{3}c\right)\right]$

$\beta = 1-\beta(\mu) = \Phi\left[\frac{5}{3}(c+\mu_0-\mu)\right] - \Phi\left[\frac{5}{3}(-c+\mu_0-\mu)\right]$

$\alpha \nearrow \Rightarrow c \nearrow \Rightarrow \beta \downarrow$.

$\square$.

由 扫描全能王 扫描创建

5.7 ① 对均值检验

$H_0: \mu = 10.5 \longleftrightarrow H_1: \mu \neq 10.5$

$T(x) = \frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma} \sim N(0,1)$

$|T(x)| = \left|\frac{\sqrt{15}(10.48-10.5)}{0.15}\right| = 0.5164 < u_{0.025} = 1.96$

∴ 接受 H_0.

② 对方差检验

$H_0: \sigma^2 = 0.15^2 \longleftrightarrow H_1: \sigma^2 \neq 0.15^2$

检验统计量 $T = \frac{(n-1)S^2}{\sigma_0^2} = 39.8444$

接受域 $\bar{D} = \{(x_1, \cdots, x_n);\ \chi^2_{n-1}(1-\frac{\alpha}{2}) \leq T \leq \chi^2_{n-1}(\frac{\alpha}{2})\}$

$\chi^2_{14}(0.975) = 5.629 \qquad \chi^2_{14}(0.025) = 26.119$

∴ $\chi^2 \notin \bar{D}$. 拒绝原假设 H_0.

综上 工作不正常.

□

5.8 考虑 $H_0: \mu \leq 1600 \longleftrightarrow H_1: \mu > 1600$

$\sigma = 150,\ n = 26,\ \mu = 1637.\quad \alpha = 0.05$

检验统计量 $U = \frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma} = 1.2578$

拒绝域 $D = \{(x_1, \cdots, x_n);\ U > u_{0.05} = 1.645\}$

∴ 接受 H_0 在 $\alpha = 0.05$ 时.

□.