Lecture 07. Lidar Simulation: Application of Lidar Equation

- Review physical processes
- Review lidar equation and its picture
- How to start lidar simulation?
- Lidar parameters and atmosphere parameters
- Simulation of resonance fluorescence return
- Simulation of Rayleigh scattering return
- Summary
Review Physical Processes

- Interaction between light and objects
  1. Scattering (instantaneous elastic & inelastic): Mie, Rayleigh, Raman
  2. Absorption and differential absorption
  3. Laser induced fluorescence
  4. Resonance fluorescence
  5. Doppler shift and Doppler broadening
  6. Boltzmann distribution
  7. Reflection from target or surface

- Light propagation in atmosphere or medium: transmission vs extinction

  Extinction = Scattering + Absorption

\[
T(\lambda, R) = \exp\left[-\int_0^R \alpha(\lambda, r) dr\right]
\]

\[
\alpha(\lambda, R) = \sum_i \left[ \sigma_{i,\text{ext}}(\lambda) n_i(R) \right]
\]
Scattering in Atmosphere

Pure Rotational Raman

Rayleigh

Cabannes Line

Stokes VRR

Stokes RR anti-Stokes

anti-Stokes VRR
Absorption and Fluorescence

Absorption Line

Lidar

ON
OFF

Energy

Absorption
Fluorescence radiation
Vibrational relaxation
Elastic scattering
Internal conversion and collisions

Intensity
Wavelength
# Backscatter Cross-Section Comparison

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<th>Physical Process</th>
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| Mie (Aerosol) Scattering              | $10^{-8} - 10^{-10}\text{ cm}^2\text{sr}^{-1}$ | Two-photon process  
Elastic scattering, instantaneous                                                   |
| Resonance Fluorescence                | $10^{-13}\text{ cm}^2\text{sr}^{-1}$ | Two single-photon process (absorption and spontaneous emission)  
Delayed (radiative lifetime)                                                       |
| Molecular Absorption                  | $10^{-19}\text{ cm}^2\text{sr}^{-1}$ | Single-photon process                                                   |
| Fluorescence from molecule, liquid, solid | $10^{-19}\text{ cm}^2\text{sr}^{-1}$ | Two single-photon process  
Inelastic scattering, delayed (lifetime)                                           |
| Rayleigh Scattering                   | $10^{-27}\text{ cm}^2\text{sr}^{-1}$ | Two-photon process  
Elastic scattering, instantaneous                                                   |
| Raman Scattering                      | $10^{-30}\text{ cm}^2\text{sr}^{-1}$ | Two-photon process  
Inelastic scattering, instantaneous                                                   |
Physical Process

- Elastic Scattering by Aerosols and Clouds
- Absorption by Atoms and Molecules
- Inelastic Scattering
- Elastic Scattering By Air Molecules
- Resonance Scattering/Fluorescence By Atoms
- Doppler Shift
- Laser Induced Fluorescence
- Reflection from Surfaces

Device

- Mie Lidar
- DIAL
- Raman Lidar
- Rayleigh Lidar
- Resonance Fluorescence Lidar
- Wind Lidar
- Fluorescence Lidar
- Target Lidar Laser Altimeter

Objective

- Aerosols, Clouds: Geometry, Thickness
- Gaseous Pollutants
- Ozone
- Humidity (H₂O)
- Aerosols, Clouds: Optical Density
- Temperature in Lower Atmosphere
- Stratos & Mesos Density & Temp
- Temperature, Wind Density, Clouds in Mid-Upper Atmos
- Wind, Turbulence
- Marine, Vegetation
- Topography, Target
Review Lidar Equation

- General lidar equation with angular scattering coefficient
  \[ N_S(\lambda, R) = N_L(\lambda_L) \cdot \left[ \beta(\lambda, \lambda_L, \theta, R) \Delta R \right] \cdot \frac{A}{R^2} \cdot \left[ T(\lambda_L, R)T(\lambda, R) \right] \cdot \left[ \eta(\lambda, \lambda_L)G(R) \right] + N_B \]

- General lidar equation with total scattering coefficient
  \[ N_S(\lambda, R) = N_L(\lambda_L) \cdot \left[ \beta_T(\lambda, \lambda_L, R) \Delta R \right] \cdot \frac{A}{4\pi R^2} \cdot \left[ T(\lambda_L, R)T(\lambda, R) \right] \cdot \left[ \eta(\lambda, \lambda_L)G(R) \right] + N_B \]

- General lidar equation in angular scattering coefficient \( \beta \) and extinction coefficient \( \alpha \) form
  \[ N_S(\lambda, R) = \left[ \frac{P_L(\lambda_L) \Delta t}{hc/\lambda_L} \right] \cdot \left[ \beta(\lambda, \lambda_L, \theta, R) \Delta R \right] \cdot \left( \frac{A}{R^2} \right) \cdot \exp \left[ -\int_0^R \alpha(\lambda_L, r') dr' \right] \cdot \exp \left[ -\int_0^R \alpha(\lambda, r') dr' \right] \cdot \left[ \eta(\lambda, \lambda_L)G(R) \right] + N_B \]
Specific Lidar Equations

- **Lidar equation for Rayleigh lidar**

\[
N_S(\lambda, R) = \left( \frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) \left( \beta(\lambda, R) \Delta R \right) \left( \frac{A}{R^2} \right) T^2(\lambda, R) (\eta(\lambda) G(R)) + N_B
\]

- **Lidar equation for resonance fluorescence lidar**

\[
N_S(\lambda, R) = \left( \frac{P_L(\lambda) \Delta t}{hc/\lambda} \right) \left( \sigma_{\text{eff}}(\lambda, R)n_c(z) R_B(\lambda) \Delta R \right) \left( \frac{A}{4\pi R^2} \right) \left( T_{\text{a}}^2(\lambda, R) T_{\text{c}}^2(\lambda, R) \right) (\eta(\lambda) G(R)) + N_B
\]

- **Lidar equation for differential absorption lidar**

\[
N_S(\lambda_{\text{on}}^{\text{off}}, R) = N_L(\lambda_{\text{on}}^{\text{off}}) \left[ \beta_{\text{sca}}(\lambda_{\text{on}}^{\text{off}}, R) \Delta R \right] \left( \frac{A}{R^2} \right) \exp \left[ -2 \int_0^z \alpha(\lambda_{\text{on}}^{\text{off}}, r') dr' \right] \\
\times \exp \left[ -2 \int_0^z \sigma_{\text{abs}}(\lambda_{\text{on}}^{\text{off}}, r') n_c(r') dr' \right] \left[ \eta(\lambda_{\text{on}}^{\text{off}}) G(R) \right] + N_B
\]
How to Start Lidar Simulation?

- Lidar simulation of return signals is a direct application of lidar equation involving physical processes.
- Let us start with the Arecibo K Doppler lidar since the system and atmosphere parameters have been estimated quite well. Let us start with MatLab code or other equivalent code.
- 1st, write down all fundamental constants used in lidar.
- 2nd, gather lidar parameters & atmosphere parameters.
- 3rd, start with the laser source of transmitter and follow the lidar picture from transmitted photons, through atmosphere transmission, backscatter probability, collection probability, and receiver efficiency, to detected photon numbers.
- 4th, understand the physical process of light interaction with objects to calculate the backscatter probability.
- 5th, get the final results and verify them with reality.
Lidar Parameters

- Lidar parameters for lidar simulation
- Laser pulse energy, repetition rate, pulse duration,
- Laser central wavelength, linewidth, chirp
- Laser divergence angle
- Transmitter mirror reflectivity
- Telescope primary mirror diameter and reflectivity
- Telescope/receiver field-of-view (FOV)
- Receiver mirrors’ transmission,
- Fiber coupling efficiency, transmission
- Filter peak transmission, bandwidth, out-of-band rejection
- Detector quantum efficiency and maximum count rate.
Atomic and Atmosphere Parameters

- Atomic parameters for lidar simulation
  1) Atomic energy level structure, degeneracy
  2) Spontaneous transition rate $A_{ki}$, oscillator strength $f$
  3) Atomic mass or molecular weight
  4) Resonance frequency or wavelength
  5) Isotope shift, abundance, line strength

- Atmosphere parameters for lidar simulation
  1) Lower atmosphere transmission
  2) Atmosphere number density
  3) Atmosphere pressure and temperature
  4) Species number density or column abundance
  5) Background sky radiance, solar angle, base altitude, etc.
Simulation of Resonance Fluorescence

- Besides common issues in lidar simulation, the main point in simulation of resonance fluorescence return is to correctly estimate the effective cross section and column abundance / density of these atomic species, e.g., K.
- Effective scattering cross section can be affected by laser central frequency, linewidth, saturation, optical pumping, branching ratio, Hanle effect, etc.
- Correct estimate of this involves comprehensive understanding of the physical process and spectroscopy knowledge - This is why spectroscopy class is important!
- Column abundance and density vary with season, latitude, and are also affected by waves etc. Usually we use a mean column abundance as a representative.
Simulation of Rayleigh Return

- This is relatively simpler compared to resonance fluorescence, because the Rayleigh scatter is straightforward.
- The key is to correctly estimate the Rayleigh backscatter cross section and atmosphere number density.
- Usually atmosphere number density can be taken from standard US atmosphere or MSIS model. MSIS number density varies with season and location.
- Estimate of atmosphere transmission or extinction is also very important. If going to lower atmosphere, it could be tricky as scattering from aerosols may interfere the lidar return signals.
Rayleigh Backscatter Cross Section

- It is common in lidar field to calculate the Rayleigh backscatter cross section using the following equation

\[
\frac{d\sigma_m(\lambda)}{d\Omega} = 5.45 \cdot \left(\frac{550}{\lambda}\right)^4 \times 10^{-32} \left( m^2 sr^{-1} \right)
\]

where \(\lambda\) is the wavelength in nm.

- The Rayleigh backscatter cross section can also be estimated from the Rayleigh backscatter coefficient

\[
\beta_{\text{Rayleigh}}(\lambda, z, \theta = \pi) = 2.938 \times 10^{-32} \frac{P(z)}{T(z)} \cdot \frac{1}{\lambda^{4.0117}} \left( m^{-1} sr^{-1} \right)
\]

where \(\lambda\) is the wavelength in meter, \(P\) in mbar, \(T\) in Kelvin.

\[
\therefore \frac{d\sigma_m(\lambda)}{d\Omega} = \frac{\beta_{\text{Rayleigh}}(\lambda, z, \pi)}{n_{\text{atmos}}(z)} \left( m^2 sr^{-1} \right)
\]
Summary

- To understand physical processes precisely is the key to successful lidar estimate and applications.

- Lidar equation may change form to best fit for each particular physical process and lidar application.

- Our first step of lidar simulation is to apply the lidar equation directly to estimate lidar returns of resonance fluorescence and Rayleigh scattering from the atmosphere based on Arecibo K Doppler lidar.

- This first HWK project will be the first step, and more projects will be added onto it.